The equation for the capsize of ships in seas with high winds and periodic beam waves has been shown to be modeled by a one-well potential oscillator with a saddle. Rewriting this equation in the form

$$\dot{x} - y = 0$$

$$\dot{y} - x + x^2 = \varepsilon[-\beta' y + F' \sin \omega t]$$

assume that $\boldsymbol{\varepsilon}$ is small and derive the criteria using the Melnikov function. As a hint, note that the unperturbed Hamiltonian is given by

$$H = \tfrac{1}{2}\dot{y}^2 - \frac{x^2}{2} + \frac{x^3}{3}$$

and that the unperturbed homoclinic orbit (i.e., the orbit that goes through the saddle when $\varepsilon = 0$) is given in parametric form

$$x_h(t) = 1 - \frac{3}{1 + \cosh t}$$

$$y_h(t) = \frac{3 \sinh t}{(1 + \cosh t)^2}$$

the Melnikov function can be written as two integrals on the line $(-\infty < t < \infty)$ $\boldsymbol{I_1}$**, ,** $\boldsymbol{I_2}$**.** The value of $\boldsymbol{I_1}$, is 2/15, while the value of the second $\boldsymbol{I_2}$**,** is solved using the calculus of complex variables and the method of residues. Plot the critical value of forcing amplitude versus frequency and show that the lowest values occur at a value of $\omega < 1$.