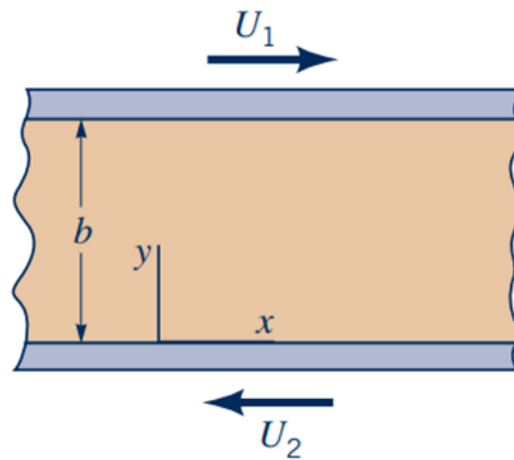




- 1- An incompressible, viscous fluid is placed between horizontal, infinite, parallel plates as is shown in the figure. The two plates move in opposite directions with constant velocities,  $U_1$  and  $U_2$ , as shown. The pressure gradient in the  $x$  direction is zero, and the only body force is due to the fluid weight. Use the Navier–Stokes equations to derive an expression for the velocity distribution between the plates. Assume laminar flow.

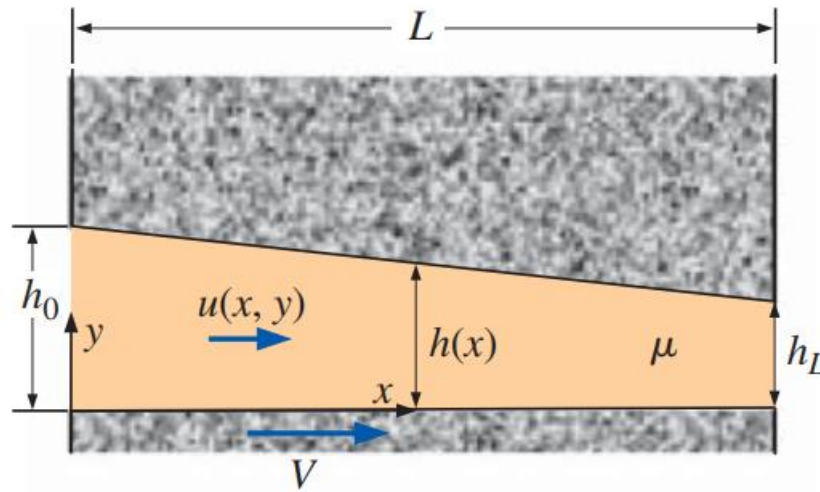


- 2- A *slipper-pad bearing* is often encountered in lubrication problems. Oil flows between two blocks; the upper one is stationary, and the lower one is moving in this case. The thin gap between the blocks converges with increasing  $x$ . Specifically, gap height  $h$  decreases linearly from  $h_0$  at  $x = 0$  to  $h_L$  at  $x = L$ . The pressure  $P$  varies nonlinearly from  $P = P_0$  at  $x = 0$  to  $P = P_L$  at  $x = L$ . ( $\frac{\partial p}{\partial x}$  is not constant). Gravity forces are negligible in this flow field, which we approximate as two-dimensional, steady, and laminar.

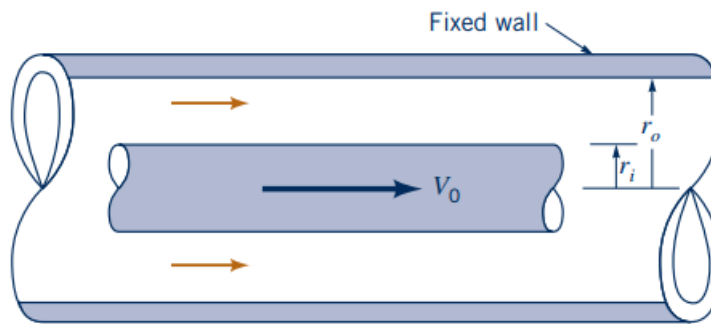
(a) List appropriate boundary conditions on  $u$ .

(b) Solve the creeping flow approximation of the  $x$ -momentum equation to obtain an expression for  $u$  as a function of  $y$  (and indirectly as a function of  $x$  through  $h$  and  $dP/dx$ , which are functions of  $x$ ).

You may assume that  $P$  is *not* a function of  $y$ . Your final expression should be written as  $u(x, y) = f(y, h, dP/dx, V, m)$ .



- 3- An incompressible Newtonian fluid flows steadily between two infinitely long, concentric cylinders as shown in figure. The outer cylinder is fixed, but the inner cylinder moves with a longitudinal velocity  $V_0$  as shown. The pressure gradient in the axial direction is  $-\Delta p/L$ . For what value of  $V_0$  will the drag on the inner cylinder be zero? Assume that the flow is laminar, axisymmetric, and fully developed.



- 4- Consider two-dimensional, incompressible, steady Couette flow (flow between two infinite parallel plates with the upper plate moving at constant speed and the lower plate stationary). Let the fluid be *non-Newtonian*, with its viscous stresses given by

$$\tau_{xx} = a \left( \frac{\partial u}{\partial x} \right)^c \quad \tau_{yy} = a \left( \frac{\partial v}{\partial y} \right)^c \quad \tau_{zz} = a \left( \frac{\partial w}{\partial z} \right)^c$$

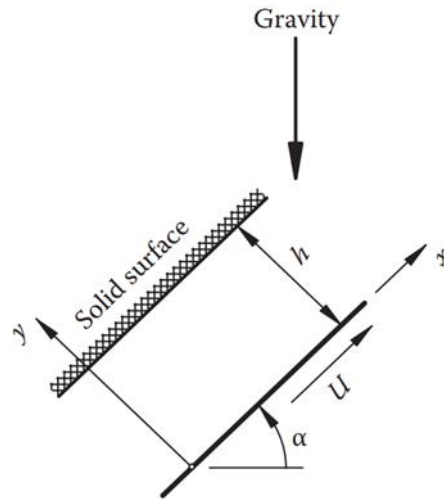
$$\tau_{xy} = \tau_{yx} = \frac{1}{2} a \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^c \quad \tau_{xz} = \tau_{zx} = \frac{1}{2} a \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^c \quad \tau_{yz} = \tau_{zy} = \frac{1}{2} a \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^c$$

where  $a$  and  $c$  are constants of the fluid. Neglect gravity and pressure gradient.

(a) Find the velocity profile  $u(y)$ .

(b) How does the velocity profile for this case compare to that of a Newtonian fluid?

- 5- A moving belt is inclined at an angle  $\alpha$  to the horizontal. The lower end of this belt is immersed in a pool of liquid, and the belt drags some of the liquid with it as it moves upward and out of the liquid. The liquid may be assumed to be viscous but incompressible. Using the configuration shown in the following figure, solve the Navier–Stokes equations for the following quantities:



- (a) The velocity distribution in the liquid layer
- (b) The volumetric flow rate of liquid in the  $x$  direction per unit width
- (c) The angle  $\alpha$  for which the volumetric flow rate is zero