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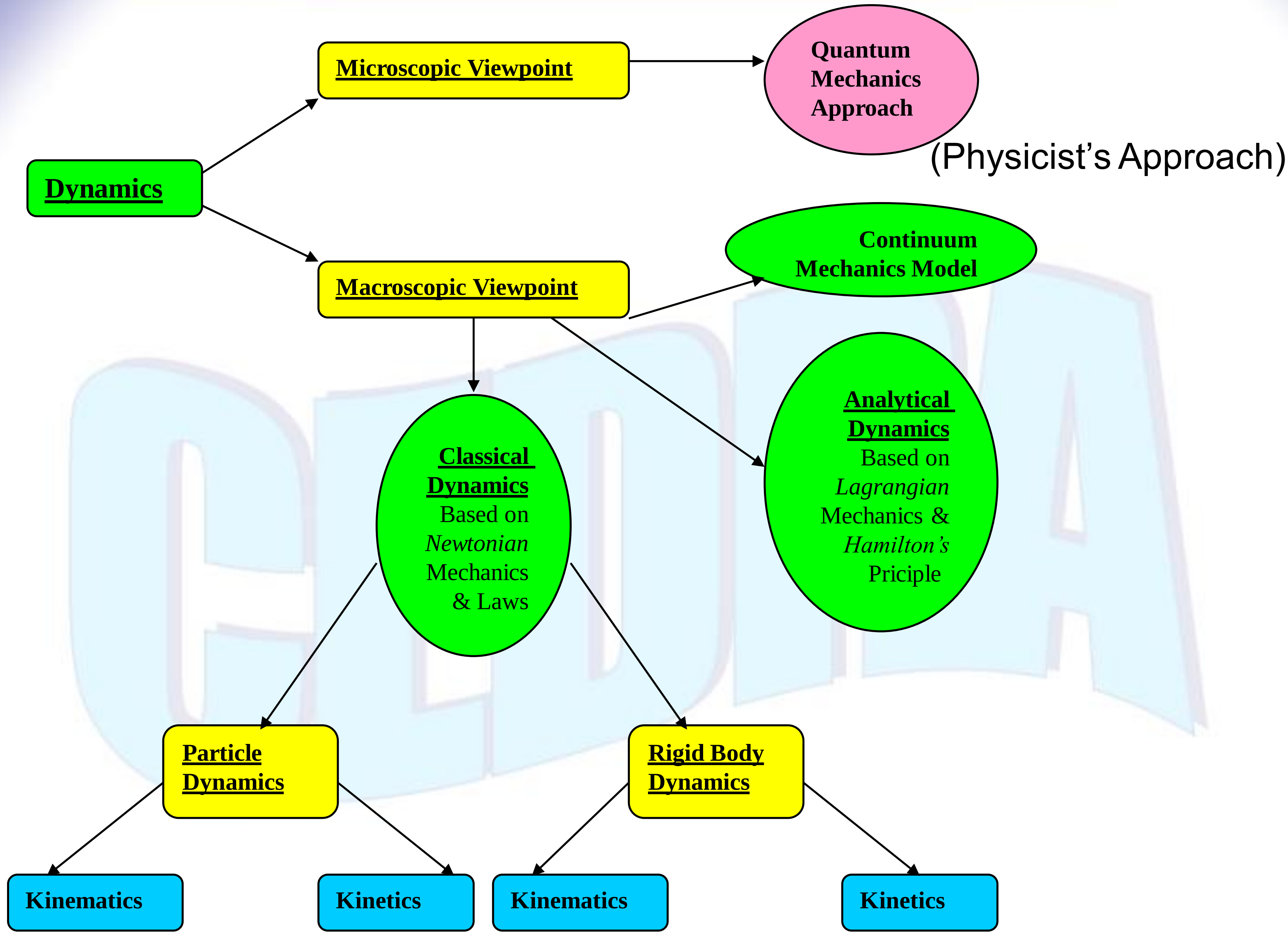
BASIC CONCEPTS

A Quick Review of Undergraduate Dynamics

***Mechanics*: is one of the oldest and basic branches of Physics. Mechanics deals with the related ideas of Force, Energy, Inertia, and Motion.**

***Dynamics*: is that branch of mechanics that is concerned with the forces acting on an object and its motion produced by the force system (i.e., is the relationship between forces acting on a system and the motion of that system.**


Dynamics
Microscopic Viewpoint
Quantum Mechanics Approach
(Physicist's Approach)
Macroscopic Viewpoint
Continuum Mechanics Model
Classical Dynamics Based on *Newtonian* Mechanics & Laws
Analytical Dynamics Based on *Lagrangian* Mechanics & *Hamilton's* Priciple
Particle Dynamics
Rigid Body Dynamics
Kinematics
Kinetics
Kinematics
Kinetics

***Kinematics*: is only concerned with the geometry of motion and deals with relationships among displacement, velocity, acceleration, and time without regard to forces causing the motion.

***Knietics*: is concerned with the force analysis of bodies in motion (force, mass, and acceleration).

SOME BASIC DEFINITIONS

Particle: is a small portion of matter that its dimensions are negligible in the analysis of a physical problem.

Rigid Body: is an aggregate of particles, of which the distance between any pair **remains constant** throughout the dynamic process.

Deformable Body: is an aggregate of particles, of which the distance between any pair **may change** throughout the dynamic process.

***Position*:** of a particle at a given time is the point of space occupied by the particle at that time.

***Body Point*:** is a point fixed in the rigid body or on its imaginary extension throughout the motion.

***Reference Frame*:** is a frame in which the kinematical and kinetic properties of a system can be defined, (i.e., *Newtonian* Reference Frame).

Note: Do not confuse Reference Frames with Coordinate Systems (i.e. Cartesian, Cylindrical). Many coordinate systems can be embedded in a given reference frame. (any rigid body may be regarded as a reference frame).

***NEWTONIAN MECHANICS*:** In utilizing the Newton's laws of motion, we must define and consider;

***Newtonian Reference Frame* "NRF" (Inertial Reference Frame, or Galilean Reference Frame):** is defined as a reference frame (coordinate) that does not rotate and whose origin is either fixed in space or if it translates, then it moves in a straight line at a constant velocity. **In other words:** *it is a **Non-Accelerating** and **Irrotational** reference frame*.

The earth, which is often used as a reference frame, is rotating about its own axis and this axis, in turn, is revolving about the sun. The solar system, consisting of the sun and its planets, is a small part of the vast *Milky Way galaxy* which is revolving in space. **Note that: *NRF* does not consider the motion of the *solar system* "sun & its planets" within the Milky Way galaxy or the motion of this galaxy within the universe**.

Newtonian Laws:

1st Law (law of Inertia): A particle will remain at rest or moves with constant speed along a straight line, unless it is acted upon by a resultant force.

2nd Law (Law of Motion): A particle acted upon by an unbalanced force " **F** " receives an acceleration " **a** " that is in the direction of the force and has a magnitude which is directly proportional to the force.

$$\boxed{\underline{F} = m\underline{a}} \qquad (2.1)$$

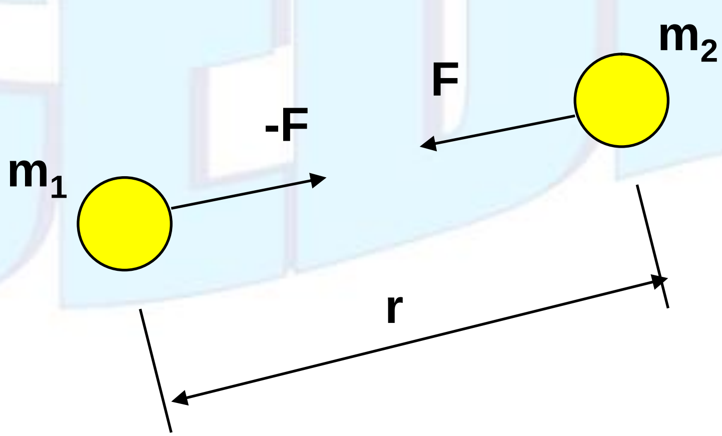
3rd Law (Law of Mutual Interaction): For every force acting on a particle, the particle exerts an equal, opposite, and collinear reactive force.

4th Law (Law of Gravitation): Two particles of mass $\mathbf{m_1}$ and $\mathbf{m_2}$, mutually attract each other with equal and opposite forces, **F** and **–F**, whose magnitude is:

Force of Attraction = $\mathbf{F = (Gm_1m_2)/r^2}$ **(2.2)**

r = Distance between two particles

G = *Universal Gravitation Constant* = $\mathbf{6.673 \times 10^{-11}\ m^3/kg.s^2}$ **(from exp. Data)**



Then, ***Force of Attraction*** between **earth** with mass **M** and a body of mass **m** is:

$$F = (GMm)/r^2 \qquad (2.3)$$

Where; **M= 5.976 × 1024 kg**, and $\mathbf{r = r_e = 6371\ km}$.

Therefore, *Weight (gravitational attraction)* of a body near the earth's surface during a free-fall in vacuum is:

$$W = m\,(GM/r^2) = mg \qquad (2.4)$$

$g = GM/r^2 =$ ***gravitational acceleration*** $= 9.807\ m/s^2 = 32.17\ ft/s^2$ **(2.5)**

***Force*:** A force is best described by the way that a person feels, sensually or otherwise.

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KINEMATICAL QUANTITIES:

Position Vector: a vector describing the position of a point or particle at a time " **t** " in a coordinate system.

$\underline{\mathbf{r}}_{\mathbf{p/o}}(\mathbf{t})$ = ***Position Vector*** : {1st kinematical quantity}

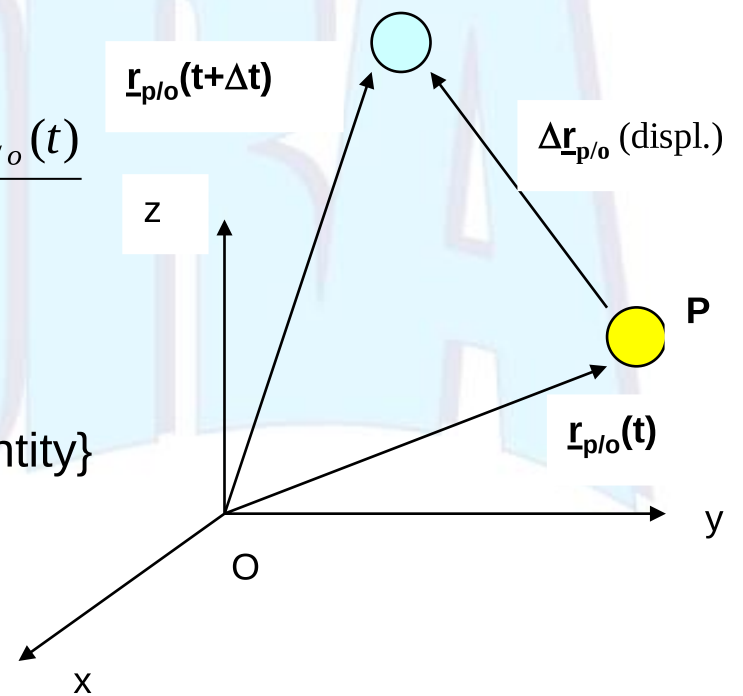
Velocity Vector:

$$\underline{\mathbf{V}}_{\mathbf{p}} = \lim_{\Delta t \to 0} \frac{\Delta \underline{r}_{p/o}}{\Delta t} = \lim \frac{\underline{r}_{p/o}(t+\Delta t) - \underline{r}_{p/o}(t)}{\Delta t}$$

$$\underline{\mathbf{V}}_{\mathbf{p}} = \frac{d\underline{r}_{p/o}}{dt} = \dot{\underline{r}}_{p/o} \qquad \mathbf{(2.6)}$$

{2nd kinematical quantity}

$$\mathbf{V}_{\mathbf{p}} = \left| \underline{v}_{P} \right| = \textit{Speed}$$



KINEMATICAL QUANTITIES:

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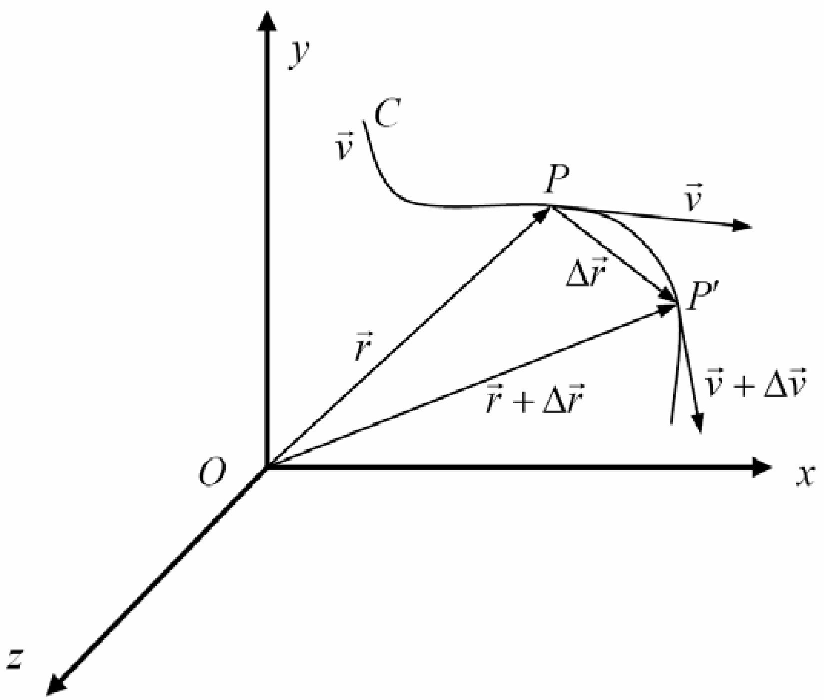
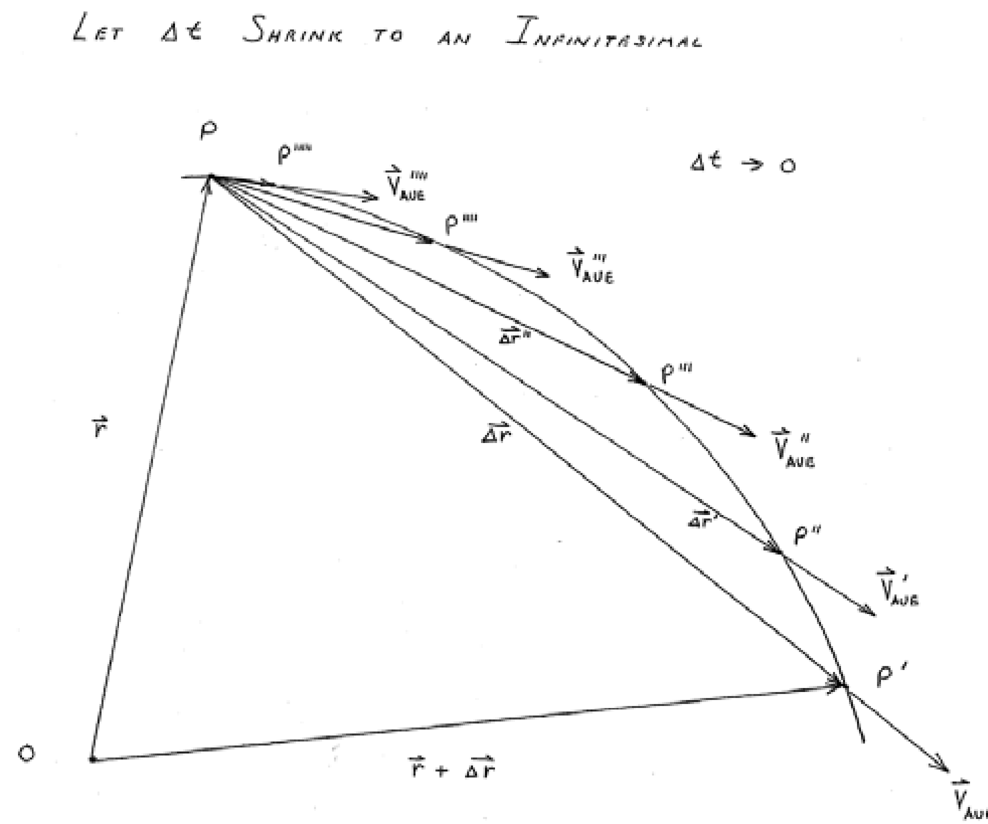
Position Vector: $\underline{r}_{p/o}(t)$

Velocity Vector: $\underline{V}_p = \lim_{\Delta t \to 0} \dfrac{\Delta \underline{r}_{p/o}}{\Delta t} = \lim \dfrac{\underline{r}_{p/o}(t+\Delta t) - \underline{r}_{p/o}(t)}{\Delta t}$





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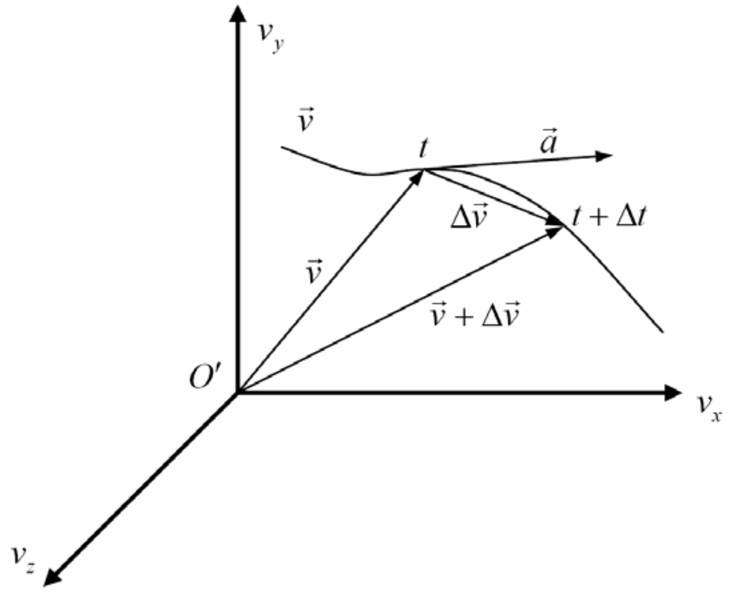
Acceleration Vector:

$$\underline{a}_p = \frac{d\underline{v}_p}{dt} = \dot{\underline{v}}_p = \ddot{\underline{r}}_{p/o}$$ {3rd kinematical quantity} **(2.7)**

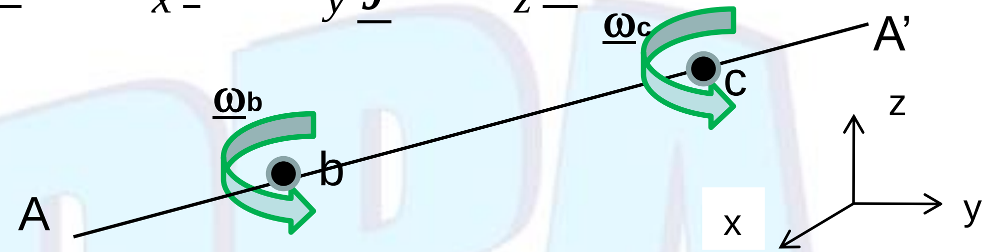


Jerk Vector: (*sudden movement*) is the rate of change of acceleration. It is occasionally considered.

$$\underline{J}_p = \frac{d\underline{a}_p}{dt} = \dot{\underline{a}}_p = \ddot{\underline{v}}_p = \dddot{\underline{r}}_{p/o}$$ {4th kinematical quantity} **(2.8)**

Angular Velocity: of a line-segment is equal to the time rate of change of its angular position.

$$\underline{\omega} = \lim_{\Delta t \to 0} \frac{\Delta \underline{\theta}}{\Delta t} = \frac{d\underline{\theta}}{dt} = \underline{\dot{\theta}} = \omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k} \qquad \textbf{(2.9)}$$



$\underline{\omega}$ is a ***free vector*** : $\underline{\omega}_b = \underline{\omega}_c$

Angular Acceleration: of a line-segment is equal to the time rate of change of its angular velocity.

$$\underline{\alpha} = \frac{d\underline{\omega}}{dt} = \underline{\dot{\omega}} = \underline{\ddot{\theta}} \qquad \textbf{(2.10)}$$

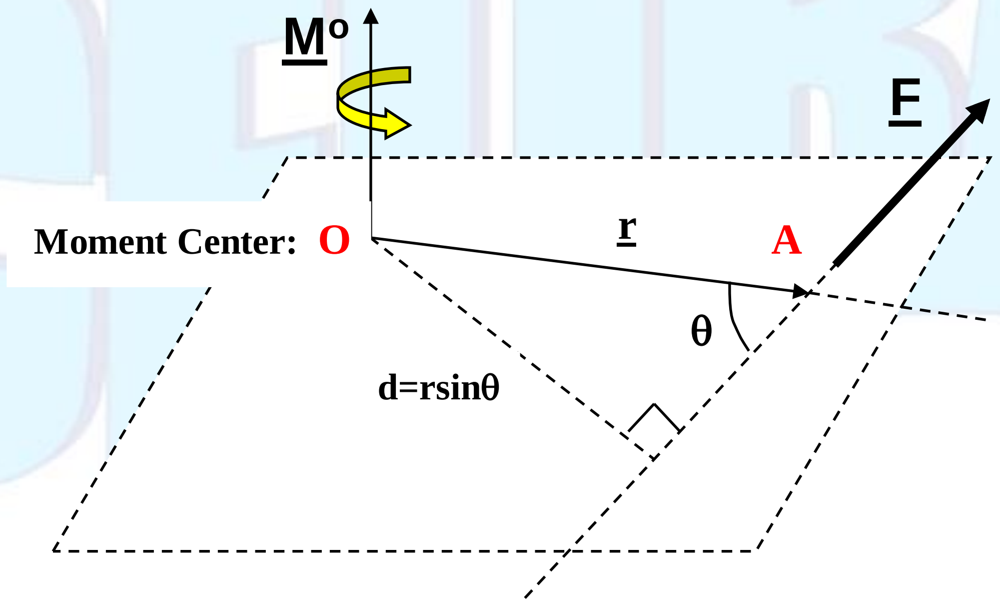
Moment: is the effect of force " $\underline{\mathbf{F}}$ " about any given point in space called the "*moment center*".

$$\underline{M}^{o} = \underline{r} \times \underline{F} \qquad \textbf{(2.11)}$$

$\underline{M}^o$ = *Moment Vector* of the force **$\underline{F}$** about the moment center "**O**".

$\underline{r}$ = *Vector Moment Arm*; vector drawn from point "**O**" to any point like "**A**" on the line of action of the force **$\underline{F}$**.

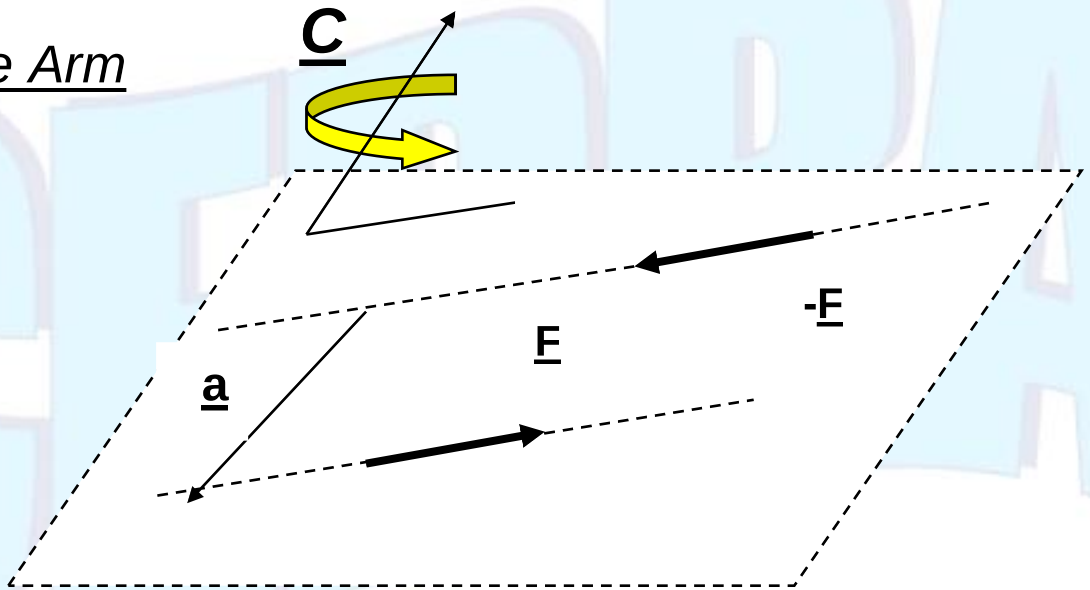
$$\mathbf{M^o} = \left|\underline{M}^o\right| = \left|\underline{r}\right|\left|\underline{F}\right|\sin\theta = Fr\sin\theta = Fd \qquad \{\textit{magnitude}\text{ of } \underline{\mathbf{M}}^o\} \qquad \mathbf{(2.12)}$$



Theorem-1: The moment of an equal and opposite pair of forces is called the ***Couple*** " **C** ", and is invariant to the position of the moment center.

$$\underline{C} = \underline{a} \times \underline{F} \qquad (2.13)$$

a = *Vector Couple Arm*



(**a**, **F**, -**F** are *coplanar*, and **C** is ⊥ to the plane)

Theorem-2: The simplest equipollent system to a force system is *one force* and *one couple*, called the ***resultant system***.

Therefore, for a system of N-force {$\underline{\mathbf{F}}_i$} we have:

Resultant Force:

$$\underline{F}_R = \sum_{i=1}^{N} \underline{F}_i \tag{2.14}$$

(line-of-action of "$\underline{\mathbf{F}}_\mathbf{R}$" passes through "**O**", since all "$\underline{\mathbf{F}}_\mathbf{i}$"s are written with respect to "**O**").

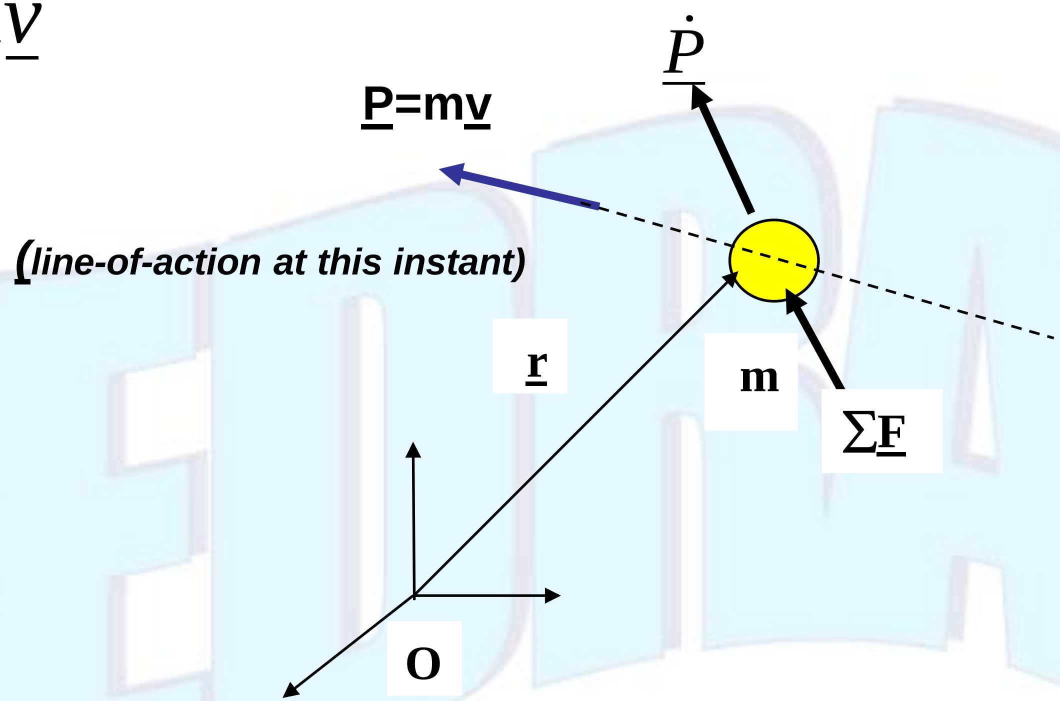
Resultant Couple:

$$\underline{C}_R = \underline{M}^o = \sum_{i=1}^{N} \underline{r}_i \times \underline{F}_i \tag{2.15}$$

{$\underline{\mathbf{r}}_\mathbf{i}$} = set of moment arms of **{$\underline{\mathbf{F}}_\mathbf{i}$}** with respect to the moment center "**O**"

Linear-Momentum (Momentum): of a particle is defined by the ***mass*** times the ***velocity*** of the particle.

$$\underline{P} = m\underline{v} \tag{2.16}$$



Moment of Momentum (Angular-Momentum): of a particle about a point "O" is defined as:

$$\underline{H}^{o} = \underline{r} \times \underline{P} \tag{2.17}$$

Newton's 2nd Law: may now be re-stated as "*resultant of all forces applied on a particle is equal to the time rate of change of its momentum*".

$$\underline{F} = m\underline{a} = m\dot{\underline{v}} = \frac{d}{dt}(m\underline{v}) = \dot{\underline{P}} \qquad \Rightarrow \qquad \underline{F} = \dot{\underline{P}} \qquad \textbf{(2.18)}$$

TIME DERIVATIVE OF A VECTOR:

Theorem-3: For a general vector like **$\underline{A}$ = A$\underline{e}_A$** which contains the time variation of its magnitude and direction, we have:

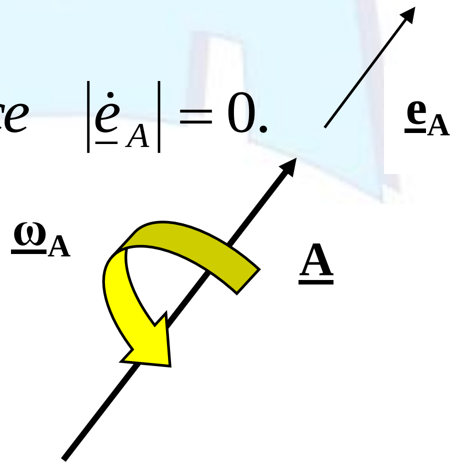
$$\dot{\underline{A}} = \dot{A}\underline{e}_A + \underline{\omega}_A \times \underline{A}$$

{*Jaumann Rate of a Vector*}

(2.19)

Proof:

$\dot{\underline{A}} = \dot{A}\underline{e}_A + A\dot{\underline{e}}_A$, *where*: $\dot{\underline{e}}_A = \underline{\omega}_A \times \underline{e}_A$, $\sin ce$ $\left|\dot{\underline{e}}_A\right| = 0.$



Theorem-4: The time derivative of a vector **A** in terms of its *Cartesian* component set {$\mathbf{A_i}$}, where; $\underline{A} = A_i(t)\underline{e}_i(t)$, is:

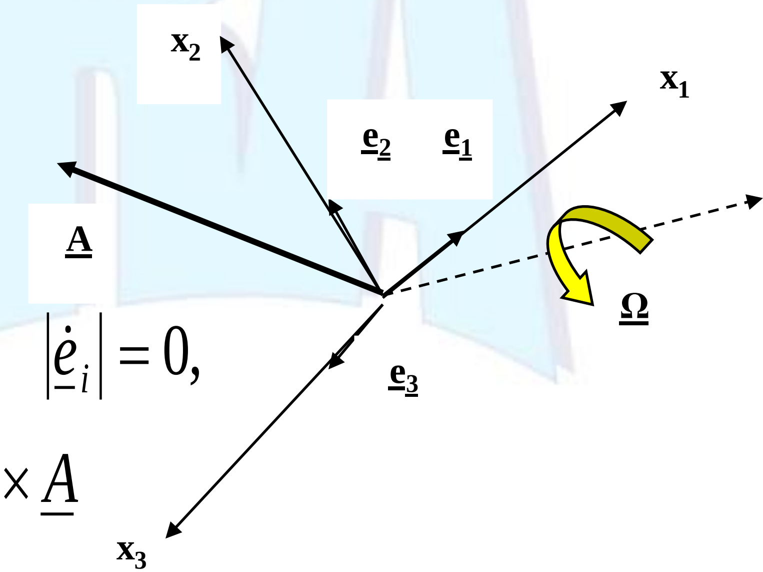
$$\dot{\underline{A}} = \dot{A}_i \underline{e}_i + \underline{\Omega} \times \underline{A} \qquad \textbf{(2.20)}$$

$\underline{\Omega}$ = angular velocity of the Cartesian coordinate {x_i}.

First Term: observed/relative change of the vector in the reference frame.

Second Term: change in **A** due to rotation of the coordinates.

Proof:



$$\dot{\underline{A}} = \dot{A}_i \underline{e}_i + A_i \dot{\underline{e}}_i \quad , \quad where: \ \dot{\underline{e}}_i = \underline{\Omega} \times \underline{e}_i , \quad since \ \left|\dot{\underline{e}}_i\right| = 0,$$

$$\dot{\underline{A}} = \dot{A}_i \underline{e}_i + A_i(\underline{\Omega} \times \underline{e}_i) = \dot{A}_i \underline{e}_i + \underline{\Omega} \times A_i \underline{e}_i = \dot{A}_i \underline{e}_i + \underline{\Omega} \times \underline{A}$$

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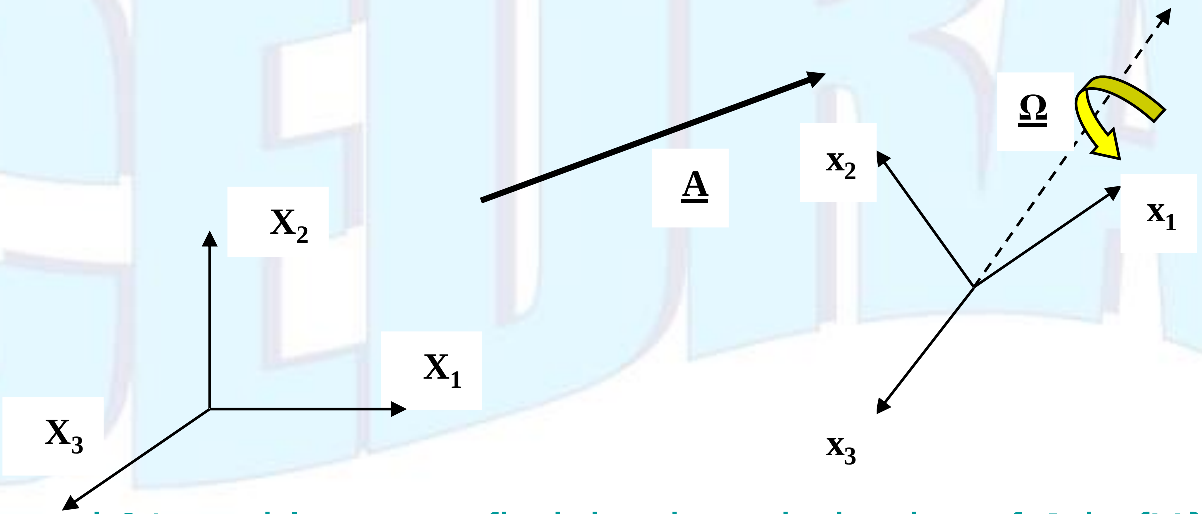
Theorem-4 (another version): The time derivative of any vector $\underline{\mathbf{A}}$ in *two frames* $\{x_i\}$ and $\{X_i\}$ are related as follows:

$$\left(\frac{d\underline{A}}{dt}\right)_{X_1X_2X_3} = \left(\frac{d\underline{A}}{dt}\right)_{x_1x_2x_3} + \underline{\Omega} \times \underline{A} \qquad \textbf{(2.21)}$$

$\underline{\Omega}$ = angular velocity of the Cartesian coordinate $\{x_i\}$ in $\{X_i\}$.



(Equations 20 and 21 enable one to find the time-derivative of $\underline{\mathbf{A}}$ in $\{X_i\}$ without having to resolve $\underline{\mathbf{A}}$ into components parallel to unit vectors fixed in $\{X_i\}$).

DIMENSIONS AND UNITS:

The *units* specified for the measurement of physical quantities are defined to be consistent with the *Newon's 2nd Law*: **F = ma**

For a *freely-falling* object in the vacuum, *Newton's 2nd law*, becomes:

$$\mathbf{W = mg} \tag{2.22}$$

Where; **F = W (weight), a =g (acceleration of gravity).**

Note: For a *unit-mass* "m", the *weight* of the object is *g-units of force*. (i.e. Weight = (1) g = g-units of force)

In general there are two basic systems of units:

L-M-T (SI, System International) System*:** The ***three fundamental units of this system are:

L = length

M = m = mass

T = time

{If *L* is in *Centimeters*, *M* is in *Grams*, and *T* is in *Seconds*, then the ***unit of Force*** from *Newton's 2nd law* will be in ***dyne*** (gr.cm/s^2)}.

***Dyne*:** a dyne is defined as the force necessary to accelerate **1** gram of mass at a rate of **1** cm/s^2.

Ex: Since, g = 980.66 cm/s^2 on the earth, therefore, **1** gram mass weighs about:

W = mg = (1 gr.)(980.66 cm/s^2) = 980.66 *dynes*.

{If *L* is in *meters*, *M* is in *Kilograms*, and *T* is in *Seconds*, then the ***unit of Force*** from *Newton's 2nd law* will be in ***Newton*** (kg.m/s^2)}.

***Newton*:** a Newton is defined as the force necessary to accelerate **1** kilogram of mass at a rate of **1** m/s^2.

Ex: Since, g = 9.807 m/s^2 on the earth, therefore, **1** kilogram mass weighs about:

W = mg = (1 kg)(9.807 m/s^2) = 9.807 *Newtons*.

L-F-T (British/US Customary System) System:

The ***three fundamental units*** of this system are:

L = length

F = Force (instead of mass M)

T = time

{If *L* is in *foot*, *F* is in *pounds*, and *T* is in *Seconds*, then the ***unit of Mass*** from *Newton's 2nd law*, "M=m=W/g", will be in ***slug*** (lb.s^2/ft)}.

Slug: **1** pound is the force necessary to accelerate **1** slug of mass at a rate of **1** ft/s^2.

Ex: Since, g = 32.17 ft/s^2 on the earth, therefore, **1** slug mass weighs about: W = mg = (1 slug)(32.17 ft/s^2) = 32.17 lbs.

{1 lb = 4.46 Newton, 1 Slug =14.63 kg}

Dimensions: The fundamental units of **L-M-T** or **L-F-T** systems can be used to represent any physical quantities in mechanics.

When a physical quantity is represented by the fundamental units, the resulting expression is called the *dimensional* form of that quantity.

Ex: Dimensional form of **Velocity** is **[LT^{-1}]**, and that of **Acceleration** is **[LT^{-2}]**.

Theorem-5: ***The Law of Dimensional Homogeneity***: states that physical equations must be homogeneous in dimensional sense (a necessary but not a sufficient condition for correctness of equations, since the validity of dimensionless coefficients could not be checked).

Note that: *Dimensions* and *Units* are two different terminologies. For example: the **Dimension of Length** is always equal to " **L** ", where as **Length** may be expressed in **different units** (inches, meters, foot...)

Physical Quantity	L-M-T , (SI) System	L-F-T, (USCS/British) System
Length, L	L	L
Force, F	MLT^{-2}	F
Mass, M, m	M	$FL^{-1}T^{2}$
Time, t	T	T
Linear Velocity, v	LT^{-1}	LT^{-1}
Linear Acceleration, a	LT^{-2}	LT^{-2}
Angle, θ radians	Dimensionless	Dimensionless
Angular Velocity, ω	T^{-1}	T^{-1}
Angular Acceleration, α	T^{-2}	T^{-2}
Moment, M^{o}	$ML^{2}T^{-2}$	FL
Linear Momentum, P	MLT^{-1}	FT
Angular Momentum, H^{o}	$ML^{2}T^{-1}$	FLT
Mass Moment of Inertia, I	ML^{2}	FLT^{2}
Area Moment of Inertia, J	L^{4}	L^{4}
Work or Energy, W or E	$ML^{2}T^{-2}$	FL
Power $\dot{W}$	$ML^{2}T^{-3}$	FLT^{-1}
Area, A	L^{2}	L^{2}
Volume, V	L^{3}	L^{3}
Stress, σ	$ML^{-1}T^{-2}$	FL^{-2}
Modulus of Elasticity, E	$ML^{-1}T^{-2}$	FL^{-2}
Mass Density, γ	ML^{-3}	$FL^{-4}T^{2}$

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