
بسم الله الرحمن الرحيم

In the Name of Allah

Advanced Engineering Dynamics

Ali Meghdari, Ph.D., Professor

Email: meghdari@sharif.edu
Home Page: http://meghdari.sharif.edu

Sundays & Tuesdays: 16:30-17:45

OFFICE HOURS: Sundays: 14:30-16:00,
Tel: 6616-5541

- **TEXT BOOK: Advanced Engineering Dynamics**, By: Jerry H. Ginsberg, Cambridge University Press, 2nd Ed., 1995, Electronics Version 2008, and Lecture Notes.
- **REFERENCES:**

Engineering Mechanics: Dynamics, By: J.L. Meriam & L.G. Kraige, John-Wiley & Sons, 4th Ed., 1998.

Advanced Dynamics; Modeling & Analysis, By: A.F. D’Souza & V.K. Garg, Prentice-Hall, 1984.

Dynamics, By: T.R. Kane & D.A. Levinson, McGraw-Hill, 1985.

GRADING:

Homework	(10 % of the Final Grade)*
Quiz/Seminar/Project:	(30% of the Final Grade)
Mid-Term Exam	(30% of the Final Grade)
Final Exam:	(30% of the Final Grade)

* Homework will be assigned every other session, and solutions will be posted online. Short pop quizzes will be given sometimes during the semester.

Dynamic Forces:

Failing to acknowledge the importance of dynamic forces and loads can have severe consequences…

- **New Ferrari: $1,000,000**
- **Son borrows Father's new car to try out......**

Dynamic Forces:

… and hits a utility pole at 250 Km/Hour.

- **Car only had 15 Km on it…**

Dynamic Forces:

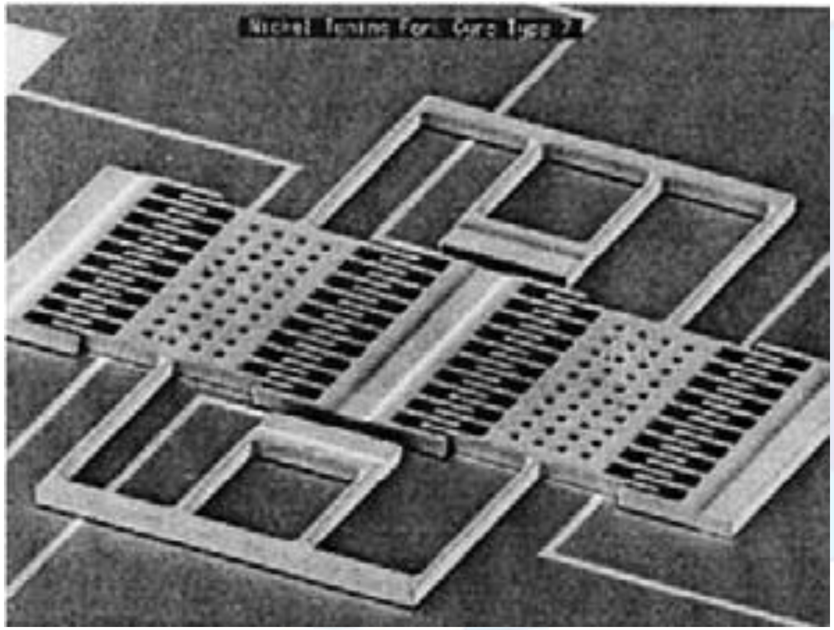
Dynamics are important in recreation as well.

Applications

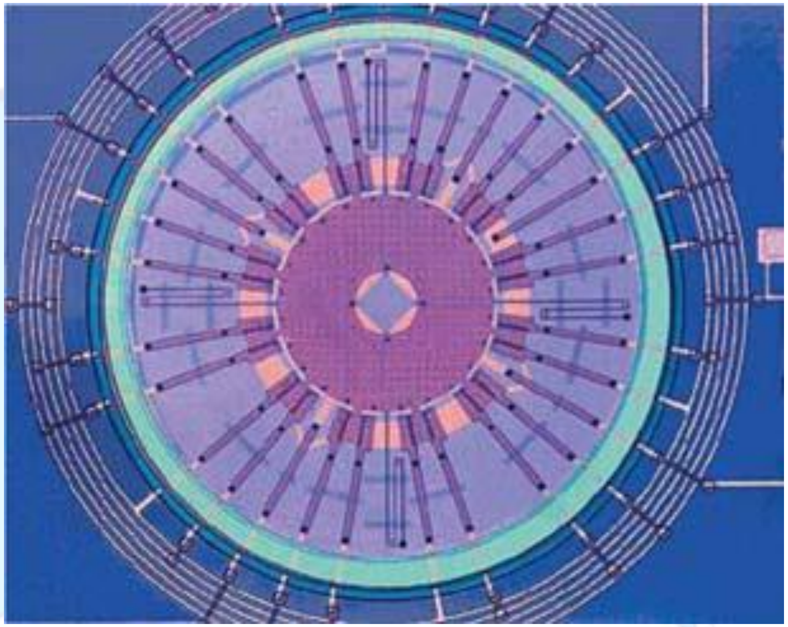
- Satellite orientation and control
- Launch vehicles
- Weapons systems

Applications

- **Micro/Nano Sensors – accelerometers, gyroscopes, rotation sensors…**



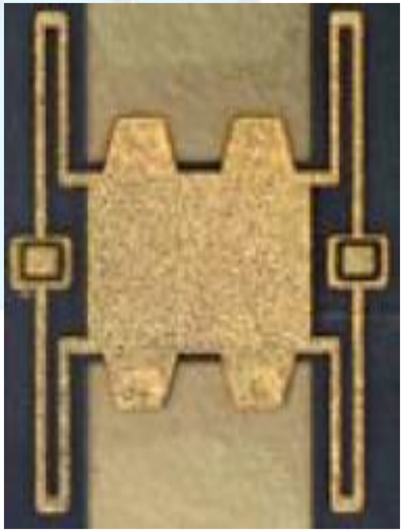
Draper Labs Comb Drive Tuning Fork Gyro



Vibrating Wheel Gyro: Berkeley Sensors and Actuators Center

Applications

- **Predicting loads on:**
  - Airplanes
  - Automobiles
  - MEMS / NANO Systems
  - Manufacturing Tools
  - Robots and Manipulators
  - Bones and Muscles
  - and just about anything…



Does water flowing down a drain spin in different directions depending on which hemisphere you're in? And if so, why?

The direction of motion is caused by the Coriolis effect, due to rotation of earth?!

TOPICS:

1. A Quick Review of Cartesian Tensors
2. Introduction, and Review of Undergraduate Dynamics
3. Kinematics: Coordinate Transformations, Curvilinear Coordinates, Generalized Coordinates, Euler's Angles, Moving Reference Frame, General 3-D Motion.
4. Particle Dynamics
5. Inertia Tensors
6. Rigid Body Dynamics: Eulerian Equations of Motion

Mid-Term Exam: **(1st week of Azar, 1397)**

7. Kinetic Principles in Non-Newtonian Reference Frame
8. Energy Principles: Leibniz Equations of Motion
9. Lagrange's Equations of Motion: (Constraints, Generalized Forces, Holonomic and Non-Holonomic Systems, etc.)
10. Hamilton's Principle
11. Introduction to Gyromechanics (if time permits)
12. Introduction to Kane's Equations of Motion (if time permits)

Final Examination: **(Finals Week)**

A Quick Review of Cartesian Tensors

Tensors: **Mathematical quantity used to describe a physical variable specified in a particular coordinate system by its components.**

Rank or Order: (i.e. ***r^{th}-order*** tensor)

In a 3-Dimensional ordinary physical space (Euclidean Space, E), the number of components of a tensor is " 3^r ", where " r " is *order* of the tensor.

- **If r=0 $\Rightarrow$ 3^0= 1-component, Tensor is "*Scalars*", (i.e. mass, speed)**
- **If r=1 $\Rightarrow$ 3^1= 3-components, Tensor is "*Vectors*", (i.e. velocity, force)**
- **If r=2 $\Rightarrow$ 3^2= 9-components, Tensor is "*Dyadics*", (i.e. stress, strain)**

Index Notation:

$\underline{A} = (A_x, A_y, A_z)$: **means vector A in (x,y,z) coordinates.**

$\underline{A} = (A_1, A_2, A_3)$: **means vector A in (x_1,x_2,x_3) coordinates.**

$$\underline{A} = A_x \underline{i} + A_y \underline{j} + A_z \underline{k} \equiv A_1 \underline{e}_1 + A_2 \underline{e}_2 + A_3 \underline{e}_3 \equiv A_i \underline{e}_i$$

Range Rule:

An unrepeated letter index is understood to take on the values 1,2,3. (*Range Index* or *Free Index* in one single term).

Ex:

$$\underline{F} = m\underline{a} \Rightarrow F_i = ma_i \Rightarrow \begin{Bmatrix} F_1 = ma_1;\ force-balance \quad in \quad x-direction. \\ F_2 = ma_2;\ force-balance \quad in \quad y-direction. \\ F_3 = ma_3;\ force-balance \quad in \quad z-direction. \end{Bmatrix}$$

Note: **Homogeneity in free indices is required. (*i.e.* $\partial_j \sigma_{ij} + f_i = 0 \qquad i, j = 1,2,3$)**

Summation Rule:

A repeated letter index (twice & no more) is understood to imply a summation over 1,2,3. It is called *Dummy or Summation Index*.

Ex:

Dot-Product of Two Vectors:

$$\underline{A}\cdot\underline{B}=A_1B_1+A_2B_2+A_3B_3=\sum_{i=1}^{3}A_iB_i=A_iB_i=A_mB_m$$

$$A_iB_iC_j=A_1B_1C_j+A_2B_2C_j+A_3B_3C_j \qquad (i,j=1,2,3)$$

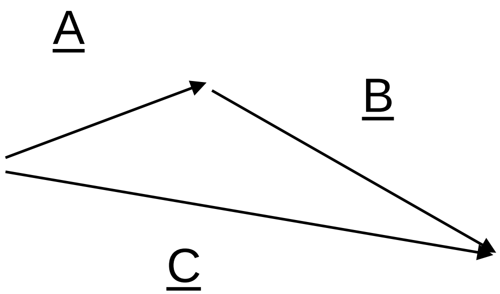
Ex:

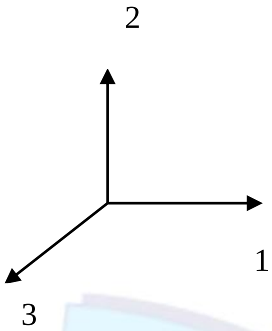
Stress Dyadic; a 2nd *order tensor* with 9-components. Consider stress at a point of a body:

$$\underline{\underline{\sigma}}=\begin{bmatrix}\sigma_{xx} & \sigma_{xy} & \sigma_{xz}\\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz}\\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz}\end{bmatrix}\equiv\begin{bmatrix}\sigma_{11} & \sigma_{12} & \sigma_{13}\\ \sigma_{21} & \sigma_{22} & \sigma_{23}\\ \sigma_{31} & \sigma_{32} & \sigma_{33}\end{bmatrix}\equiv\sigma_{ij}$$

Ex:

Vector Addition:





C = A + B

$$\begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix} = \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix} + \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \end{Bmatrix} = \begin{Bmatrix} A_1 + B_1 \\ A_2 + B_2 \\ A_3 + B_3 \end{Bmatrix} \quad \Rightarrow \quad C_k = A_k + B_k \quad (k = 1,2,3 \quad Free - Index)$$

Ex:

Cross-Product:

$$\underline{R} = \underline{S} \times \underline{T} = \begin{vmatrix} \underline{e}_1 & \underline{e}_2 & \underline{e}_3 \\ S_1 & S_2 & S_3 \\ T_1 & T_2 & T_3 \end{vmatrix} = (S_2 T_3 - S_3 T_2)\underline{e}_1 + (S_3 T_1 - S_1 T_3)\underline{e}_2 + (S_1 T_2 - S_2 T_1)\underline{e}_3$$

or;

Ex:

Cross-Product:

$$\underline{R} = \underline{S} \times \underline{T} = \begin{vmatrix} \underline{e}_1 & \underline{e}_2 & \underline{e}_3 \\ S_1 & S_2 & S_3 \\ T_1 & T_2 & T_3 \end{vmatrix} = (S_2 T_3 - S_3 T_2)\underline{e}_1 + (S_3 T_1 - S_1 T_3)\underline{e}_2 + (S_1 T_2 - S_2 T_1)\underline{e}_3$$

or;

$$\underline{R} = R_1 \underline{e}_1 + R_2 \underline{e}_2 + R_3 \underline{e}_3 = R_i \underline{e}_i = (\gamma_{ijk} S_j T_k)\underline{e}_i \qquad (i, j, k = Dummy - Indices, as: 1,2,3)$$

where:

$$R_1 \underline{e}_1 = (\gamma_{1jk} S_j T_k)\underline{e}_1 = (S_2 T_3 - S_3 T_2)\underline{e}_1 = (\gamma_{11k} S_1 T_k + \gamma_{12k} S_2 T_K + \gamma_{13k} S_3 T_k)\underline{e}_1 =$$

$$= \left(\begin{Bmatrix} \gamma_{111} S_1 T_1 + \\ \gamma_{112} S_1 T_2 + \\ \gamma_{113} S_1 T_3 \end{Bmatrix} + \begin{Bmatrix} \gamma_{121} S_2 T_1 + \\ \gamma_{122} S_2 T_2 + \\ \gamma_{123} S_2 T_3 \end{Bmatrix} + \begin{Bmatrix} \gamma_{131} S_3 T_1 + \\ \gamma_{132} S_3 T_2 + \\ \gamma_{133} S_3 T_3 \end{Bmatrix} \right) \underline{e}_1$$

R_1, R_2, R_3, all together contain *27-components*, where *3-negative*, *3-positive*, *all other 21 are zeros*.

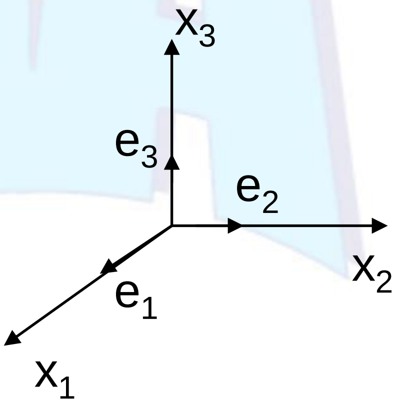
Special Tensor Quantities:

- **Permutation Symbol** $\equiv \gamma_{ijk} = \begin{Bmatrix} +1 & (i,j,k)=(1,2,3),(2,3,1),(3,1,2) \\ -1 & (i,j,k)=(3,2,1),(2,1,3),(1,3,2) \\ 0 & all-others,(\text{Re}\,peated-Indices) \end{Bmatrix}$

- **Kronecker Delta** $\equiv \delta_{mn} = \begin{Bmatrix} 1 & m=n \\ 0 & m \neq n \end{Bmatrix}$

Ex: ***Dot-Product of Unitary Orthogonal Base Vectors***:

$$\begin{Bmatrix} \underline{e}_1 . \underline{e}_1 = 1 \\ \underline{e}_1 . \underline{e}_2 = 0 \\ etc. \end{Bmatrix} \Rightarrow \quad \underline{e}_i . \underline{e}_j = \delta_{ij}$$



If $\underline{a} = a_i \underline{e}_i$ **, and** $\underline{b} = b_j \underline{e}_j$**, then:**

$$\underline{a}.\underline{b} = a_i \underline{e}_i . b_j e_j = a_i b_j \underline{e}_i . \underline{e}_j = a_i b_j \delta_{ij}$$

$$\underline{a} \cdot \underline{b} = a_i (b_j \delta_{ij}) \equiv a_i b_i$$

Ex: ***Cross-Product of Unitary Orthogonal Base Vectors***:

$$\left\{\begin{array}{l} \underline{e}_1 \times \underline{e}_1 = 0 \\ \underline{e}_1 \times \underline{e}_2 = \underline{e}_3 \\ \underline{e}_1 \times \underline{e}_3 = -\underline{e}_2 \\ etc. \end{array}\right\} \Rightarrow \quad \underline{e}_i \times \underline{e}_j = \gamma_{ijk} \underline{e}_k \quad where: \gamma_{ijk} = \left\{\begin{array}{lll} 0 & for & i = j \\ +1 & for & ijk \\ -1 & for & kji \end{array}\right\}$$

Therefore, if $\underline{a} = a_i \underline{e}_i$, and $\underline{b} = b_j \underline{e}_j$, then:

$$\underline{a} \times \underline{b} = a_i \underline{e}_i \times b_j \underline{e}_j = a_i b_j \underline{e}_i \times \underline{e}_j = a_i b_j \gamma_{ijk} \underline{e}_k \quad \Rightarrow$$

$$\underline{a} \times \underline{b} = \gamma_{ijk} a_i b_j \underline{e}_k \quad (i, j, k \rightarrow \quad dummy - indices)$$

A Vector Function: **is a vector " $\underline{r} = \underline{r}(q_j)$ " in which its magnitude and/or direction in a reference frame {A} depends on n-scalar variables " q_j " in frame {A}.**

$$\underline{r} = r_1(q_j)\underline{e}_1 + r_2(q_j)\underline{e}_2 + r_3(q_j)\underline{e}_3 = r_i(q_j)\underline{e}_i \qquad (j = 1,\ldots,n)$$

A Scalar Function: **is the coefficient of " $\underline{e}_i$ " in the vector function " $\underline{r}$ " and the reference frame {A}, being " $r_i(q_j)$ " which is also unique.**

Gradient of a Scalar Function:

Consider a Scalar Function (i.e. pressure, density) as $f(x_i)$, then:

grad f = $\nabla f = \frac{\partial f}{\partial x_1}\underline{e}_1 + \frac{\partial f}{\partial x_2}\underline{e}_2 + \frac{\partial f}{\partial x_3}\underline{e}_3 = \frac{\partial f}{\partial x_i}\underline{e}_i \qquad (\nabla \text{"Del" is a vector-operator})$

Sometimes: $\nabla = \left(\frac{\partial}{\partial x_1}\underline{e}_1 + \frac{\partial}{\partial x_2}\underline{e}_2 + \frac{\partial}{\partial x_3}\underline{e}_3\right) = \frac{\partial}{\partial x_i}\underline{e}_i = \; ,i\underline{e}_i \qquad \Rightarrow \nabla f = f,i\underline{e}_i$

Divergence of a Vector Function:

Consider a Vector Function as: $\underline{A} = A_m(x_i)\underline{e}_m$ (m=1,2,3 and i=1,2,3)

$$Div\underline{A} = \nabla\cdot\underline{A} = (\frac{\partial}{\partial x_1}\underline{e}_1 + \frac{\partial}{\partial x_2}\underline{e}_2 + \frac{\partial}{\partial x_3}\underline{e}_3)\cdot(A_1\underline{e}_1 + A_2\underline{e}_2 + A_3\underline{e}_3) =$$

$$= (\frac{\partial A_1}{\partial x_1} + \frac{\partial A_2}{\partial x_2} + \frac{\partial A_3}{\partial x_3}) = \frac{\partial A_m}{\partial x_m} = A_{m,m}$$

☆

Also:

$$\nabla\cdot\underline{A} = (\frac{\partial}{\partial x_i}\underline{e}_i)\cdot(A_m\underline{e}_m) = \frac{\partial A_m}{\partial x_i}\underline{e}_i\cdot\underline{e}_m = \frac{\partial A_m}{\partial x_i}\delta_{im} = \frac{\partial A_i}{\partial x_i} = \frac{\partial A_m}{\partial x_m} = A_{i,i} = A_{m,m}$$

Curl of a Vector Function:

Consider a Vector Function as: $\underline{V} = V_k(x_j)\underline{e}_k$ (k=1,2,3 and j=1,2,3)

$$Curl\underline{V} = \nabla\times\underline{V} = \begin{vmatrix} \underline{e}_1 & \underline{e}_2 & \underline{e}_3 \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ V_1 & V_2 & V_3 \end{vmatrix} \equiv (\frac{\partial V_3}{\partial x_2} - \frac{\partial V_2}{\partial x_3})\underline{e}_1 + (\frac{\partial V_1}{\partial x_3} - \frac{\partial V_3}{\partial x_1})\underline{e}_2 + (\frac{\partial V_2}{\partial x_1} - \frac{\partial V_1}{\partial x_2})\underline{e}_3 =$$

☆

$$= (\frac{\partial}{\partial x_j}\underline{e}_j)\times(V_k\underline{e}_k) = \frac{\partial V_k}{\partial x_j}\underline{e}_j\times\underline{e}_k \equiv \gamma_{ijk}\frac{\partial V_k}{\partial x_j}\underline{e}_i \equiv \gamma_{ijk}V_{k,j}\underline{e}_i$$

By: Professor Ali Meghdari

***Laplacian of a Scalar Function*:**

Consider a Scalar Function as f(x_j), then; the Laplacian Operator is:

$$\nabla^2 = \nabla \cdot \nabla = (\frac{\partial}{\partial x_i}\underline{e}_i)\cdot(\frac{\partial}{\partial x_j}\underline{e}_j) = \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}\delta_{ij} = \frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2} = \frac{\partial^2}{\partial x_i \partial x_i} \equiv \quad ,ii$$

Then:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x_1^2}+\frac{\partial^2 f}{\partial x_2^2}+\frac{\partial^2 f}{\partial x_3^2} = \frac{\partial^2 f}{\partial x_i \partial x_i} \equiv f,ii \qquad (i = dummy - index)$$

Example:

Show that; $\underline{A}.(\underline{B}\times\underline{C})=\underline{B}.(\underline{C}\times\underline{A})=\underline{C}.(\underline{A}\times\underline{B})$

$$let: \quad \underline{D}=\underline{B}\times\underline{C}=\gamma_{ijk}B_jC_k\underline{e}_i=D_i\underline{e}_i \Rightarrow \quad D_i=\gamma_{ijk}B_jC_k$$

$$but: \quad \underline{A}.\underline{D}=A_iD_i=A_i\gamma_{ijk}B_jC_k=\gamma_{ijk}A_iB_jC_k=B_j\gamma_{jki}C_kA_i=$$

$$=B_j(\underline{C}\times\underline{A})_j=\underline{B}.(\underline{C}\times\underline{A})$$

(it is OK to change the order of indices in a cyclic form)

Also;

$$\underline{A}.\underline{D}=A_iD_i=\gamma_{ijk}A_iB_jC_k=C_k\gamma_{kij}A_iB_j=C_k(\underline{A}\times\underline{B})_k=\underline{C}.(\underline{A}\times\underline{B})$$

متشکرم