بسم الله الرحمن الرحیم

By: Professor Ali Meghdari

NON-NEWTONIAN REFERRENCE FRAME

***Purpose*:**

- **To Study Mechanics in *Non-Newtonian Reference Frame* (NNRF).**

***Topics*:**

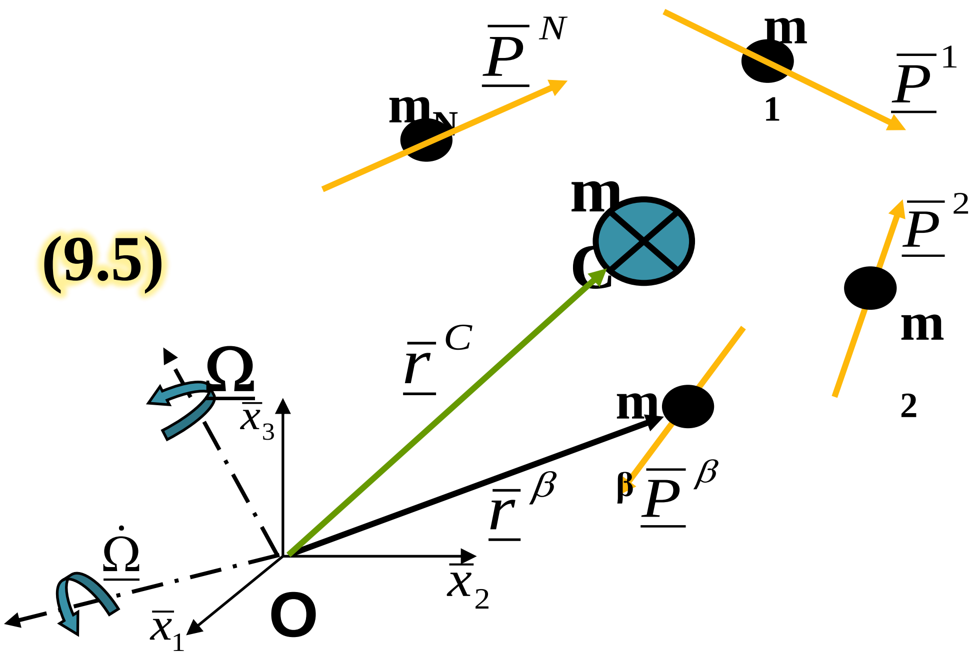
- **Kinetic Principles of a *Particle* in a *NNRF*.**
- **Kinetic Principles of a *System of Particles* & *Rigid Bodies* in NNRF.**

Kinetic Principles in a NNRF for a System of *Particles* *& Rigid Bodies*

P-Principle for a System of Particles: similar to P-Principle in NRF, the equation relates the Global Admissible Forces to the change in Kinetic State of the *mass center*.

$$\underline{\overline{P}} = m\underline{\overline{v}}^{C} = \sum_{\beta=1}^{N} \underline{\overline{P}}^{\beta}$$

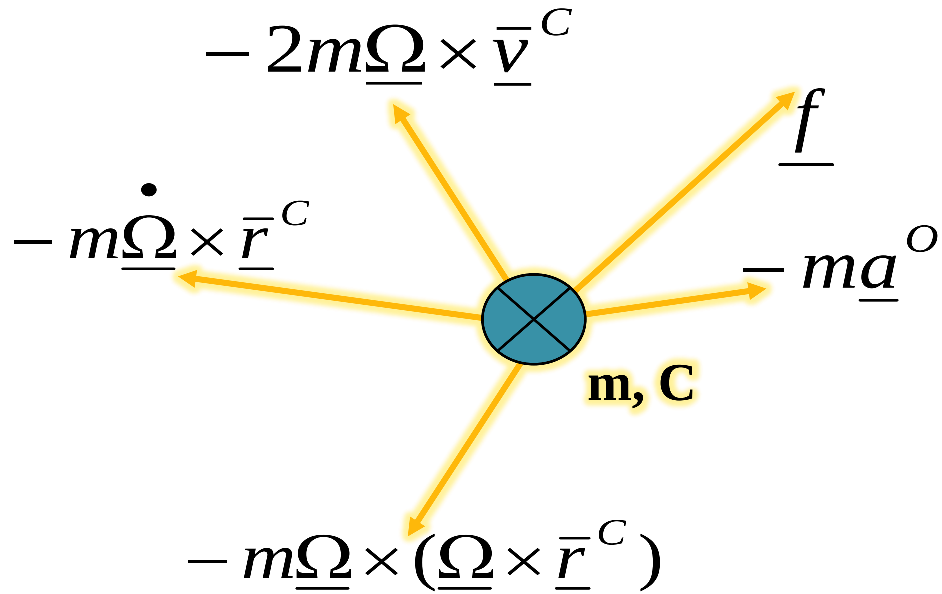
$$\underline{\overline{f}} = \frac{d}{dt}(\underline{\overline{P}}) = m\underline{\overline{a}}^{C} \qquad (9.5)$$



The *Admissible Forces* are:

$$\underline{\bar{f}} = \frac{d}{dt}(\underline{\bar{P}}) = m\underline{\bar{a}}^{C}$$

$$\underline{\bar{f}} = \underline{f} - m\underline{a}^{O} - m\underline{\dot{\Omega}} \times \underline{\bar{r}}^{C} - 2m\underline{\Omega} \times \underline{\bar{v}}^{C} - m\underline{\Omega} \times (\underline{\Omega} \times \underline{\bar{r}}^{C}) \qquad \textbf{(9.6)}$$



Note: All Global Coordinate Forces act at the mass center of the material system.

H-Principle for a System of Particles: **with respect to a moment center fixed to the NNRF, the observed time rate of change in moment-of-momentum is equal to the *totality* of (effect of) all admissible moments.**

$$\underline{\overline{H}}^{O} = \sum_{\beta=1}^{N} \underline{\bar{r}}^{\beta} \times m_{\beta} \underline{\bar{v}}^{\beta}$$ {Global Moment-of-Momentum}

$$\underline{\overline{M}}^{O} = \underline{\dot{\overline{H}}}_{i}\, \underline{u}_{i} \quad or \quad \underline{\overline{M}}^{O}_{i} = \underline{\dot{\overline{H}}}^{O}_{i} \qquad \textbf{(9.7)}$$

{Global Moment-of-Forces}

Admissible Forces on the Free Body Diagram are:

1. Newtonian Forces: $\underline{f}$

2. Non-Newtonian Forces: $\underline{\bar{f}}_{\beta} = m_{\beta}\,\underline{\bar{a}}^{\beta}$

a) Individual Particle Expression:

$$\{m_{\beta}\,\underline{a}^{O}\},$$

$$\{-m_{\beta}\,\underline{\dot{\Omega}} \times \underline{\bar{r}}^{\beta}\},$$

$$\{-m_{\beta}\,\underline{\Omega} \times (\underline{\Omega} \times \underline{\bar{r}}^{\beta})\},$$

$$\{-2m_{\beta}\,\underline{\Omega} \times \underline{\bar{v}}^{\beta}\}$$

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Admissible Forces on the Free Body Diagram are:

b) Global Expression (all particles)

$$\overline{\underline{f}} = \sum_{\beta=1}^{N} \overline{\underline{f}}_{\beta} = \sum_{\beta=1}^{N} m_{\beta} \overline{\underline{a}}^{\beta} =$$

$$= \sum_{\beta=1}^{N} \underline{f}_{\beta} - \sum_{\beta=1}^{N} m_{\beta} \underline{a}^{O} - \dot{\underline{\Omega}} \times \sum_{\beta=1}^{N} m_{\beta} \overline{\underline{r}}^{\beta} - 2\underline{\Omega} \times \sum_{\beta=1}^{N} m_{\beta} \overline{\underline{v}}^{\beta} - \underline{\Omega} \times (\underline{\Omega} \times \sum_{\beta=1}^{N} m_{\beta} \overline{\underline{r}}^{\beta})$$

$$\overline{\underline{f}} = \underline{f} - m\underline{a}^{O} - m\dot{\underline{\Omega}} \times \overline{\underline{r}}^{C} - 2m\underline{\Omega} \times \overline{\underline{v}}^{C} - m\underline{\Omega} \times (\underline{\Omega} \times \overline{\underline{r}}^{C}) \qquad (9.6)$$

And from H-Principle:

{i^th-component}

$$\overline{M}_{i}^{O} = [\sum_{\beta=1}^{N} \overline{\underline{M}}_{\beta}^{O}]_{i} = \dot{\overline{H}}_{i}^{O} \qquad (9.8)$$

***Theorem-30*:** **The moment of the Global Coordinate Inertia “ $-m\underline{a}^{O}$ ” is equal to the totality of those of individual coordinate inertia, “ $-m_{\beta}\underline{a}^{O}$ ”.**

Proof:

$$-\sum_{\beta=1}^{N}\underline{\bar{r}}^{\beta}\times m_{\beta}\underline{a}^{O}=-\underbrace{\sum_{\beta=1}^{N}m_{\beta}\underline{\bar{r}}^{\beta}}_{m\underline{\bar{r}}^{C}}\times\underline{a}^{O}=\underline{\bar{r}}^{C}\times(-m\underline{a}^{O})$$

***Theorem-31*:** **The totality (effect of all individual forces) of *individual Euler’s forces* “ $-m_{\beta}\underline{\dot{\Omega}}\times\underline{\bar{r}}^{\beta}$ ” is equivalent to the *Global Euler’s Force* “ $-m\underline{\dot{\Omega}}\times\underline{\bar{r}}^{C}$ ”, and a *Non-Newtonian Couple* “ $\underline{\bar{C}}=-\underline{\underline{\bar{I}}}^{C}\cdot\underline{\dot{\Omega}}$ ” called the *Euler’s Couple*.**

Proof:

[Total moment of individual Euler's forces about "O"]$_i$ =

$$[\sum_{\beta=1}^{N}\overline{M}_{\beta}^{O}]_i = [-\sum_{\beta=1}^{N}\underline{\overline{r}}^{\beta}\times m_{\beta}(\underline{\dot{\Omega}}\times\underline{\overline{r}}^{\beta})]_i = -\underbrace{\sum_{\beta=1}^{N} m_{\beta}[\overline{x}_k^{\beta}\overline{x}_k^{\beta}\delta_{ij} - \overline{x}_i^{\beta}\overline{x}_j^{\beta}]}_{\overline{I}_{ij}^{O}}\dot{\Omega}_j =$$

$$= -\overline{I}_{ij}^{O}\dot{\Omega}_j \qquad \textbf{(a)}$$

Similarly; [Moment of the Global Euler's forces about "O"]$_i$ =

$$\overline{M}_i^{O} = [-\underline{\overline{r}}^{C}\times m(\underline{\dot{\Omega}}\times\underline{\overline{r}}^{C})]_i = -m(\overline{x}_k^{C}\overline{x}_k^{C}\delta_{ij} - \overline{x}_i^{C}\overline{x}_j^{C})\dot{\Omega}_j \qquad \textbf{(b)}$$

The *Euler's Couple* is defined as the difference between equations (a) and (b) as:

$$\overline{C}_i = (a)-(b) = -\underbrace{[\overline{I}_{ij}^{O} - m(\overline{x}_k^{C}\overline{x}_k^{C}\delta_{ij} - \overline{x}_i^{C}\overline{x}_j^{C})]}_{\text{(by Transfer Theorem)}}\dot{\Omega}_j =$$

$$= -\overline{I}_{ij}^{C}\dot{\Omega}_j \quad \Rightarrow \quad \underline{\overline{C}} = -\underline{\underline{\overline{I}}}^{C}\cdot\underline{\dot{\Omega}} \qquad \textbf{(9.9)}$$

Remarks:

$$\underline{A}\times(\underline{B}\times\underline{C})=(\underline{A}\cdot\underline{C})\underline{B}-(\underline{A}\cdot\underline{B})\underline{C}=A_kC_k\,\underline{B}-A_jB_j\,\underline{C}$$

$$C_i=\delta_{ij}C_j$$

$$[\underline{A}\times(\underline{B}\times\underline{C})]_i=A_kC_kB_i-A_jB_jC_i=(A_kC_k\delta_{ij}-A_jC_i)B_j$$

$$Therefore:$$

$$-[\sum_{\beta=1}^{N}m_\beta\underline{\bar{r}}^{\beta}\times(\underline{\dot{\Omega}}\times\underline{\bar{r}}^{\beta})]=-\sum_{\beta=1}^{N}m_\beta[\bar{x}_k^{\beta}\bar{x}_k^{\beta}\delta_{ij}-\bar{x}_i^{\beta}\bar{x}_j^{\beta}]\dot{\Omega}_j=-\bar{I}_{ij}^{O}\dot{\Omega}_j$$

Theorem-32: **The totality effect of all individual Centrifugal Inertias, " $-m_\beta \underline{\Omega}\times(\underline{\Omega}\times\underline{\bar{r}}^{\beta})$" is equivalent to the *Global Centrifugal Inertia*, " $-m\underline{\Omega}\times(\underline{\Omega}\times\underline{\bar{r}}^{C})$", and a *Non-Newtonian Couple*, " $\underline{\bar{C}}^{*}=-\underline{\Omega}\times(\underline{\underline{\bar{I}}}^{C}\cdot\underline{\Omega})$" called the *Centrifugal Couple*.**

Proof:

Let $\underline{\bar{M}}_{\beta}^{O}$ be the moment of the centrifugal inertia of β^th^ particle about the moment center "O", then:

$$\sum_{\beta=1}^{N}\underline{\bar{M}}_{\beta}^{O}=-\sum_{\beta=1}^{N}\underline{\bar{r}}^{\beta}\times[m_\beta\underline{\Omega}\times(\underline{\Omega}\times\underline{\bar{r}}^{\beta})]=-\sum_{\beta=1}^{N}m_\beta\underline{\bar{r}}^{\beta}\times[(\underline{\Omega}\cdot\underline{\bar{r}}^{\beta})\underline{\Omega}-(\underline{\Omega}\cdot\underline{\Omega})\underline{\bar{r}}^{\beta}]=$$

$$=-\sum_{\beta=1}^{N}m_\beta(\underline{\bar{r}}^{\beta}\cdot\underline{\Omega})\underline{\bar{r}}^{\beta}\times\underline{\Omega}=-\underline{\Omega}\times\sum_{\beta=1}^{N}m_\beta[(\underline{\bar{r}}^{\beta}\cdot\underline{\bar{r}}^{\beta})\underline{\Omega}-(\underline{\bar{r}}^{\beta}\cdot\underline{\Omega})\underline{\bar{r}}^{\beta}]=$$

$$=-\underline{\Omega}\times[\sum_{\beta=1}^{N}m_\beta\underline{\bar{r}}^{\beta}\times(\underline{\Omega}\times\underline{\bar{r}}^{\beta})]=-\underline{\Omega}\times(\underline{\underline{\bar{I}}}^{O}\cdot\underline{\Omega})\qquad \textbf{(a)}$$

Similarly, the moment of the Global Centrifugal Inertia about the moment center "O" $\underline{\overline{M}}^{O}$, is:

$$\underline{\overline{M}}^{O} = -\underline{\overline{r}}^{C} \times m[\underline{\Omega}\times(\underline{\Omega}\times\underline{\overline{r}}^{C})] = -m(\underline{\Omega}\cdot\underline{\overline{r}}^{C})\underline{\overline{r}}^{C}\times\underline{\Omega} =$$

$$= -\underline{\Omega}\times m[\underline{\overline{r}}^{C}\times(\underline{\Omega}\times\underline{\overline{r}}^{C})] = -\underline{\Omega}\times \underbrace{m[\underline{\overline{x}}_k^{C}\underline{\overline{x}}_k^{C}\delta_{ij} - \underline{\overline{x}}_i^{C}\underline{\overline{x}}_j^{C}]}_{\underline{\underline{\overline{I}}}_{eq.}}\Omega_j\underline{u}_i =$$

$$= -\underline{\Omega}\times(\underline{\underline{\overline{I}}}^{O}_{eq.}\cdot\underline{\Omega}) \qquad \text{(b)}$$

Finally, the *Centrifugal Couple* is obtained as the difference between equations (a) and (b):

$$\underline{\overline{C}}^{*} = \sum_{\beta=1}^{N}\underline{\overline{M}}^{\beta} - \underline{\overline{M}}^{O} = -\underline{\Omega}\times[\underbrace{(\underline{\underline{\overline{I}}}^{O} - \underline{\underline{\overline{I}}}^{O}_{eq.})}_{\text{(from Transfer Theorem)}}\cdot\underline{\Omega}] = -\underline{\Omega}\times(\underline{\underline{\overline{I}}}^{C}\cdot\underline{\Omega}) \qquad (9.10)$$

Note that the expression is independent of the moment center "O".

***Theorem-33*:** **The equivalent system of the entire system of individual Coriolis Forces, "** $-2m_{\beta}\underline{\Omega}\times\underline{\bar{v}}^{\beta}$ **" consist of the *Global Coriolis Force*, "** $-2m\underline{\Omega}\times\underline{\bar{v}}^{C}$ **", and a *Non-Newtonian Couple*, "** $\underline{\overline{C}}^{**}=-\dot{\underline{\underline{\bar{I}}}}^{C}\cdot\underline{\Omega}-\underline{\Omega}\times\underline{\overline{H}}^{C}$ **" called the *Coriolis Couple*.**

Proof:

Let $\underline{\overline{M}}_{\beta}^{O}$ be the moment of the Coriolis force on the β[th] particle about the moment center "O", then:

$$[\sum_{\beta=1}^{N}\overline{M}_{\beta}^{O}]_i=-[\sum_{\beta=1}^{N}2m_{\beta}\underline{\bar{r}}^{\beta}\times(\underline{\Omega}\times\underline{\bar{v}}^{\beta})]_i=$$

$$-\sum_{\beta=1}^{N}m_{\beta}[\underline{\bar{r}}^{\beta}\times(\underline{\Omega}\times\underline{\bar{v}}^{\beta})+\underline{\bar{v}}^{\beta}\times(\underline{\Omega}\times\underline{\bar{r}}^{\beta})]_i-\sum_{\beta=1}^{N}m_{\beta}[\underline{\Omega}\times(\underline{\bar{r}}^{\beta}\times\underline{\bar{v}}^{\beta})]_i=$$

$$[\sum_{\beta=1}^{N}\overline{M}_{\beta}^{O}]_i = -\sum_{\beta=1}^{N} m_\beta[\bar{x}_k^\beta \dot{\bar{x}}_k^\beta \delta_{ij} - \bar{x}_i^\beta \dot{\bar{x}}_j^\beta)\Omega_j + (\dot{\bar{x}}_k^\beta \bar{x}_k^\beta \delta_{ij} - \dot{\bar{x}}_i^\beta \bar{x}_j^\beta)\Omega_j]_i -$$

$$-[\underline{\Omega} \times \sum_{\beta=1}^{N} m_\beta(\underline{\bar{r}}^\beta \times \underline{\bar{v}}^\beta)]_i =$$

$$= -\{[\frac{d}{dt}\sum_{\beta=1}^{N} m_\beta(\bar{x}_k^\beta \bar{x}_k^\beta \delta_{ij} - \bar{x}_i^\beta \bar{x}_j^\beta)\Omega_j\}_i - [\underline{\Omega} \times \underline{\overline{H}}^O]_i =$$

$$= -\{\dot{\bar{I}}_{ij}^O \Omega_j\}_i - [\underline{\Omega} \times \underline{\overline{H}}^O]_i$$

(a)

Similarly, the moment of the Global Coriolis Force can be shown to be :

$$\underline{\overline{M}}^O = -\underline{\bar{r}}^C \times 2m(\underline{\Omega} \times \underline{\bar{v}}^C) =$$

$$= -m\underline{\bar{r}}^C \times (\underline{\Omega} \times \underline{\bar{v}}^C) - m\underline{\bar{v}}^C \times (\underline{\Omega} \times \underline{\bar{r}}^C) - m\underline{\Omega} \times (\underline{\bar{r}}^C \times \underline{\bar{v}}^C) =$$

$$\underline{\overline{M}}^{O} = -\frac{d}{dt}[m(\overline{x}_k^C \overline{x}_k^C \delta_{ij} - \overline{x}_i^C \overline{x}_j^C)]\Omega_j \underline{u}_i - \underline{\Omega} \times (m\underline{\overline{r}}^C \times \underline{\overline{v}}^C)$$

(b)

Then, the *Coriolis Couple* is:

$$\underline{\overline{C}}^{**} = \sum_{\beta=1}^{N} \underline{\overline{M}}^{\beta} - \underline{\overline{M}}^{O} = -\frac{d}{dt}[\overline{I}_{ij}^{O} - m(\overline{x}_k^C \overline{x}_k^C \delta_{ij} - \overline{x}_i^C \overline{x}_j^C)]\Omega_j \underline{u}_i - \underline{\Omega} \times [\underbrace{\underline{\overline{H}}^{O} - \underline{\overline{r}}^C \times m\underline{\overline{v}}^C}_{\underline{\overline{H}}^C}] =$$

$$= -\dot{\overline{I}}_{ij}^{C} \Omega_j \underline{u}_i - \underline{\Omega} \times \underline{\overline{H}}^C = -\underline{\underline{\dot{\overline{I}}}}^C \cdot \underline{\Omega} - \underline{\Omega} \times \underline{\overline{H}}^C$$

$$\underline{\overline{C}}^{**} = -\underline{\underline{\dot{\overline{I}}}}^C \cdot \underline{\Omega} - \underline{\Omega} \times \underline{\overline{H}}^C \qquad \textbf{(9.11)}$$

Conclusion: The equivalent system of the individual coordinate forces consists of the set of *Global Coordinate Forces* (Equation 9.6) acting at the mass center, and *three Non-Newtonian Couples* (Equations:9.9, 9.10, 9.11).

***Observations*:**

1). Coordinate couples all vanish as the material system is reduced to a single particle.

2). The coordinate couples are associated with the rotational phenomena of the coordinate system. In translatory coordinates, the coordinate couples all vanish.

3). Remember that Global coordinate couples are the result of replacing individual forces by an equivalent system. Therefore, they exist only when the global force system is used.

Special Cases:

I). The *Euler's Couple* vanishes provided that the reference frame rotates at a constant angular velocity.

$$\{\underline{\bar{C}} = 0, when : \underline{\Omega} = constant\}$$

II). The ***Centrifugal Couple*** vanishes when the spin axis of the reference frame is parallel to a central principal axis of inertia of the mass system.

III). The ***Coriolis Couple*** vanishes under the following combined conditions throughout the dynamic process:

(a). The reference frame's spin axis and the relative angular velocity of the mass system are both parallel to a same central principal axis of inertia.

(b). The system is rigid.

Statement (a): $\Omega_j = \Omega\delta_{j\alpha}, \bar{I}^C_{i\alpha} = \bar{I}^C_{\alpha}\delta_{i\alpha}$ **{if x_α is the spin axis}**

Statement (b): $\overline{H}^C_k = \bar{I}^C_{\alpha}\overline{\omega}_\alpha\delta_{\alpha k}$

$$\underline{\overline{C}}^{**}_i = -\dot{\bar{I}}^C_{ij}\Omega\delta_{j\alpha} - \gamma_{ijk}\Omega\delta_{j\alpha}(\bar{I}^C_\alpha\overline{\omega}_\alpha)\delta_{\alpha k} =$$

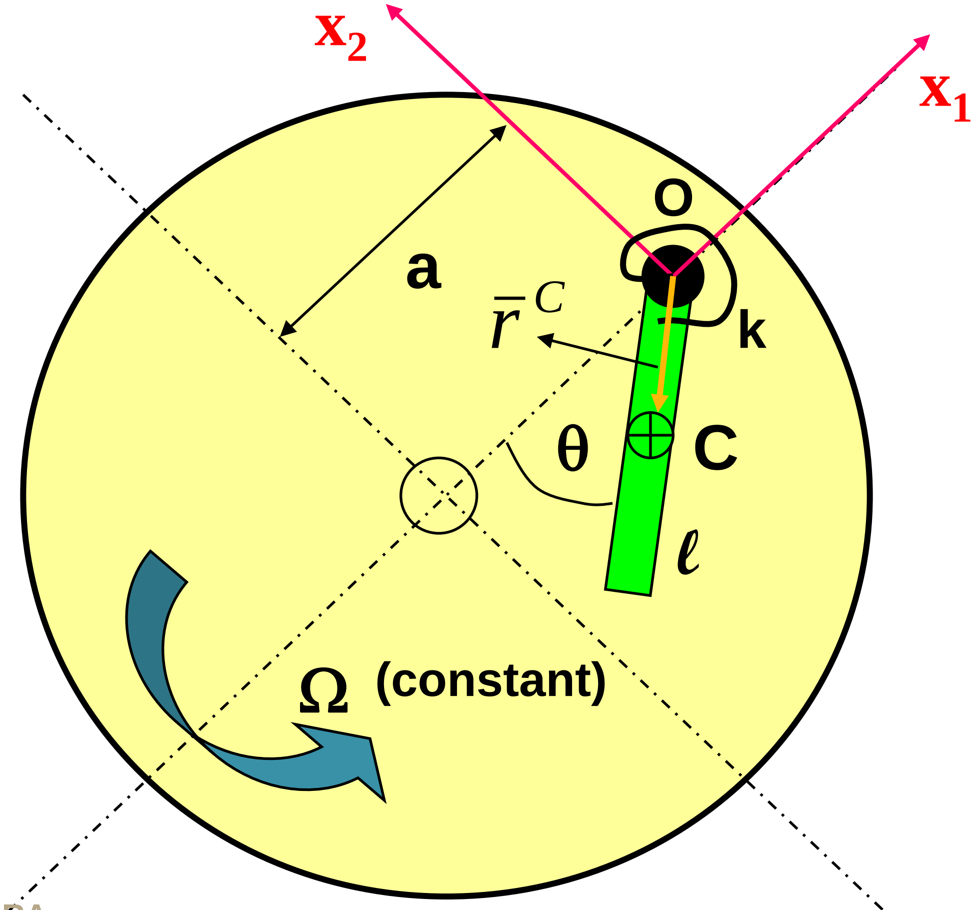
$$= -\dot{\bar{I}}^C_\alpha\Omega\delta_{i\alpha} - \gamma_{i\alpha\alpha}\Omega\bar{I}^C_\alpha\overline{\omega}_\alpha = 0 \qquad (\text{since} \quad \bar{I}^C_\alpha = \text{constant})$$

0 0

Summary:

Individual Force Set	Global Forces & Couples	Force Action
$\{-m_\beta \underline{a}^O\}$	$\{-m\underline{a}^O\}$	They all
$\{-m_\beta \dot{\underline{\Omega}} \times \underline{\bar{r}}^\beta\}$	$\{-m\dot{\underline{\Omega}} \times \underline{\bar{r}}^C, \underline{\bar{C}} = -\underline{\underline{\bar{I}}}^C \cdot \dot{\underline{\Omega}}\}$	act at
$\{-m_\beta \underline{\Omega} \times (\underline{\Omega} \times \underline{\bar{r}}^\beta)\}$	$\left\{\begin{matrix} -m\underline{\Omega} \times (\underline{\Omega} \times \underline{\bar{r}}^C) \\ \underline{\bar{C}}^* = -\underline{\Omega} \times (\underline{\underline{\bar{I}}}^C \cdot \underline{\Omega}) \end{matrix}\right\}$	the
$\{-2m_\beta \underline{\Omega} \times \underline{\bar{v}}^\beta\}$	$\left\{\begin{matrix} -2m\underline{\Omega} \times \underline{\bar{v}}^C \\ \underline{\bar{C}}^{**} = -\dot{\underline{\underline{\bar{I}}}}^C \cdot \underline{\Omega} - \underline{\Omega} \times \underline{\bar{H}}^C \end{matrix}\right\}$	Mass Center

Example: A horizontal centrifuge is spinning at a constant speed Ω. A uniform slender bar is held in equilibrium along the radial line by a torsion spring "k" at the end, which is pinned to the centrifuge at a distance "a" from the spin axis. Determine the oscillation of the bar?



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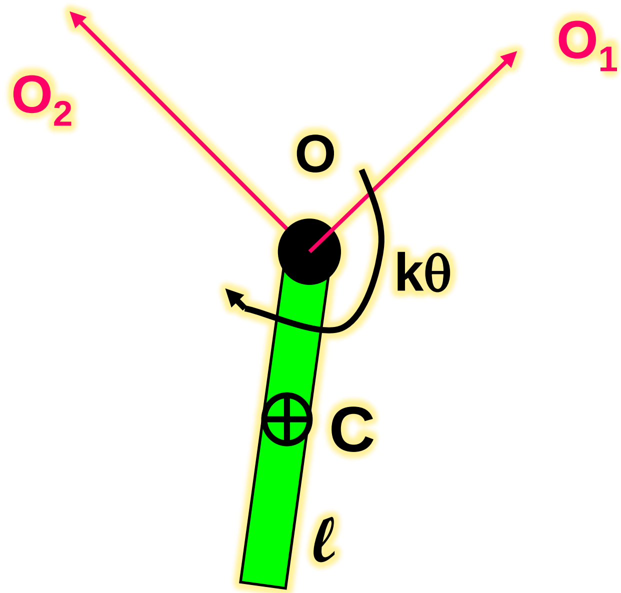
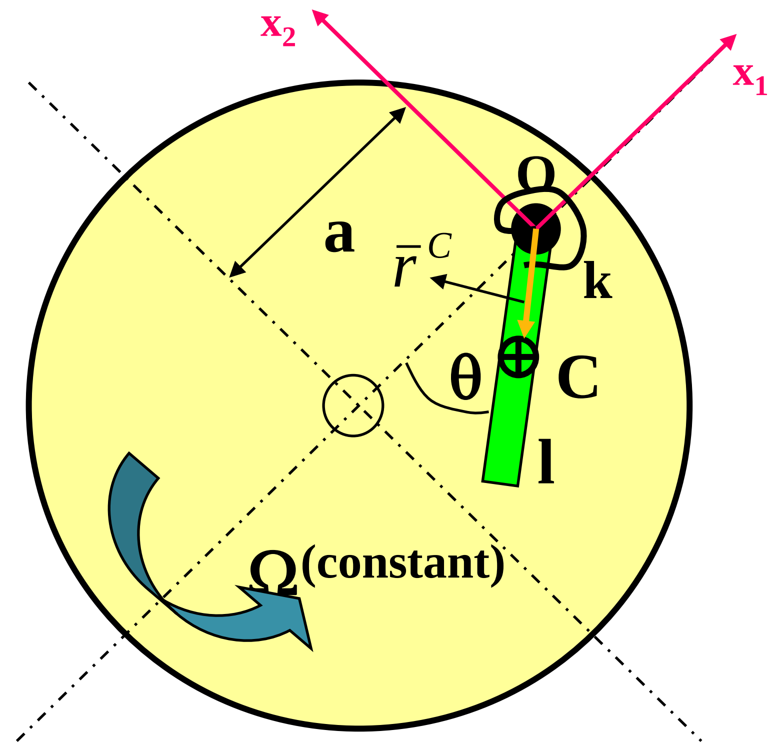
Method I: (By NRF-Earth)

1. Free Body Diagram:





2. Kinematics Analysis: Set KRF on the centrifuge with reference point at "O".

$$\underline{a}^C = \underline{a}^O + \underline{\bar{a}}^C + \dot{\underline{\Omega}} \times \underline{\bar{r}}^C + 2\underline{\Omega} \times \underline{\bar{v}}^C + \underline{\Omega} \times (\underline{\Omega} \times \underline{\bar{r}}^C)$$

$$\underline{\bar{r}}^{C} = -\frac{\ell}{2}(\cos\theta\underline{u}_1 + \sin\theta\underline{u}_2)$$

$$\underline{\bar{v}}^{C} = \frac{\ell}{2}\dot{\theta}(\sin\theta\underline{u}_1 - \cos\theta\underline{u}_2)$$

$$\underline{\bar{a}}^{C} = \frac{\ell}{2}\ddot{\theta}(\sin\theta\underline{u}_1 - \cos\theta\underline{u}_2) + \frac{\ell}{2}\dot{\theta}^2(\cos\theta\underline{u}_1 + \sin\theta\underline{u}_2)$$

$$\underline{a}^{C} = (-a\Omega^2 + \frac{\ell}{2}\ddot{\theta}\sin\theta + \frac{\ell}{2}\dot{\theta}^2\cos\theta + \frac{\ell}{2}\Omega^2\cos\theta + \ell\dot{\theta}\Omega\cos\theta)\underline{u}_1 +$$

$$(-\frac{\ell}{2}\ddot{\theta}\cos\theta + \frac{\ell}{2}\dot{\theta}^2\sin\theta + \frac{\ell}{2}\Omega^2\sin\theta + \ell\dot{\theta}\Omega\sin\theta)\underline{u}_2 \qquad \text{(a)}$$

3. **Kinetics Analysis**: **For a Rigid Body constrained in plane motion, with point "C" being an admissible moment center, we have:**

$$\begin{cases} \underline{f} = m\underline{a}^C \\ \underline{M}^C = \underline{\underline{I}}^C \underline{\alpha} \end{cases} \Rightarrow \begin{cases} O_1\underline{u}_1 + O_2\underline{u}_2 = m\underline{a}^C & \text{(b)} \\ -k\theta + O_2(\frac{\ell}{2}\cos\theta) - O_1(\frac{\ell}{2}\sin\theta) = \frac{m\ell^2}{12}\ddot{\theta} & \text{(c)} \end{cases}$$

Substitute Eq. (a) in Eq. (b), and find O_1 and O_2 , then substitute the results into Eq. (c) to have:

$$-k\theta + m[(-\frac{\ell^2}{4}\ddot{\theta}\cos^2\theta + \frac{\ell^2}{4}\dot{\theta}^2\sin\theta\cos\theta + \frac{\ell^2}{4}\Omega^2\sin\theta\cos\theta + \frac{\ell^2}{2}\dot{\theta}\Omega\sin\theta\cos\theta] +$$

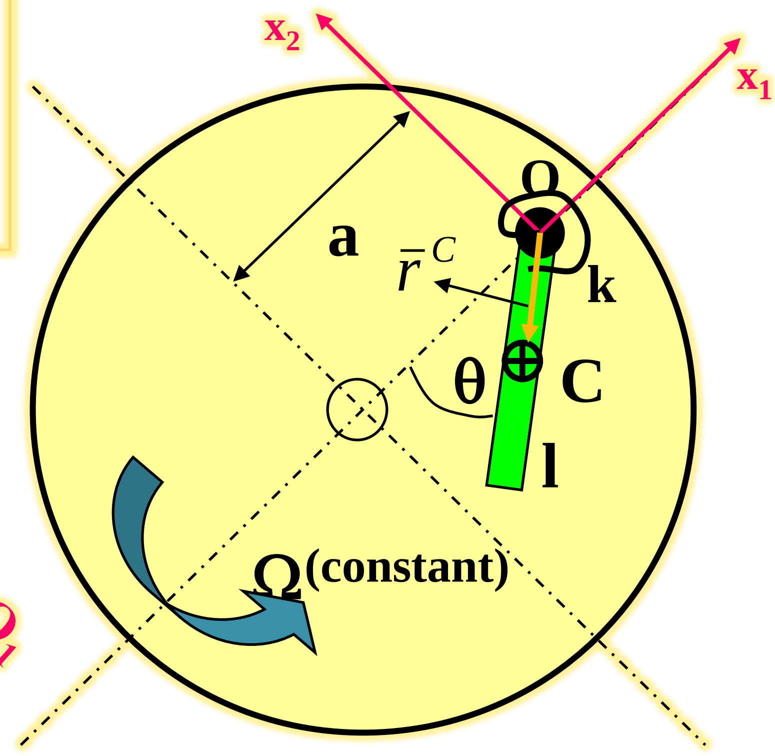
$$m[\frac{a\ell}{2}\Omega^2\sin\theta - \frac{\ell^2}{4}\ddot{\theta}\sin^2\theta - \frac{\ell^2}{4}\dot{\theta}^2\sin\theta\cos\theta - \frac{\ell^2}{4}\Omega^2\sin\theta\cos\theta -$$

$$\frac{\ell^2}{2}\Omega\dot{\theta}\sin\theta\cos\theta] = \frac{m\ell^2}{12}\ddot{\theta} \quad \Rightarrow \qquad \text{(a)}$$

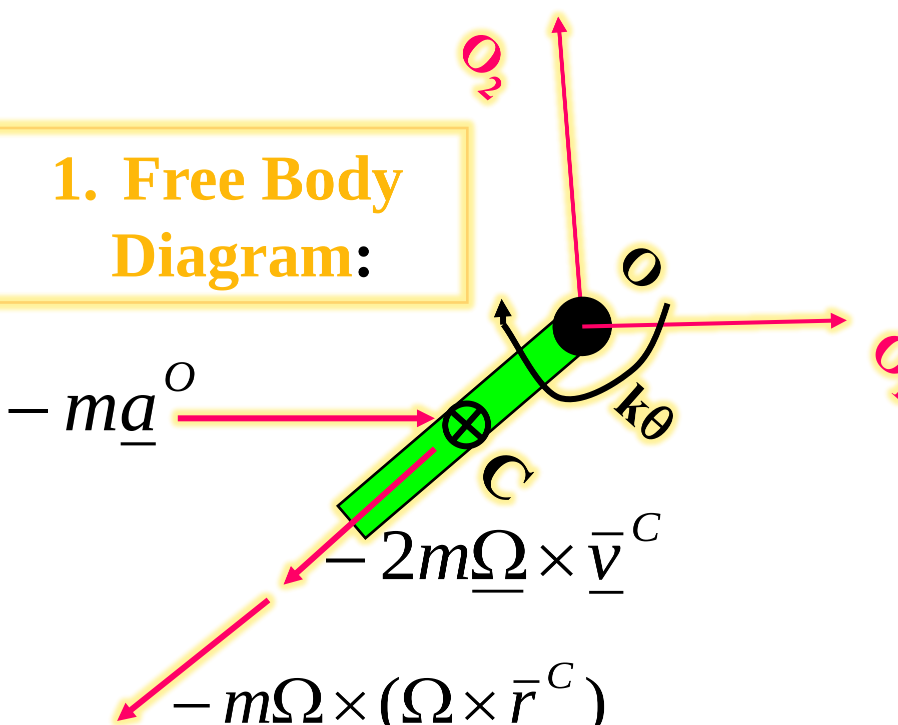
$$\ddot{\theta} - \frac{3}{2}\frac{a\Omega^2}{\ell}\sin\theta + \frac{3k}{m\ell^2}\theta = 0 \qquad (D.E.M.)$$

Method II: **(By NNRF)**

Choose NNRF as the centrifuge with reference point at "O".



1. Free Body Diagram:



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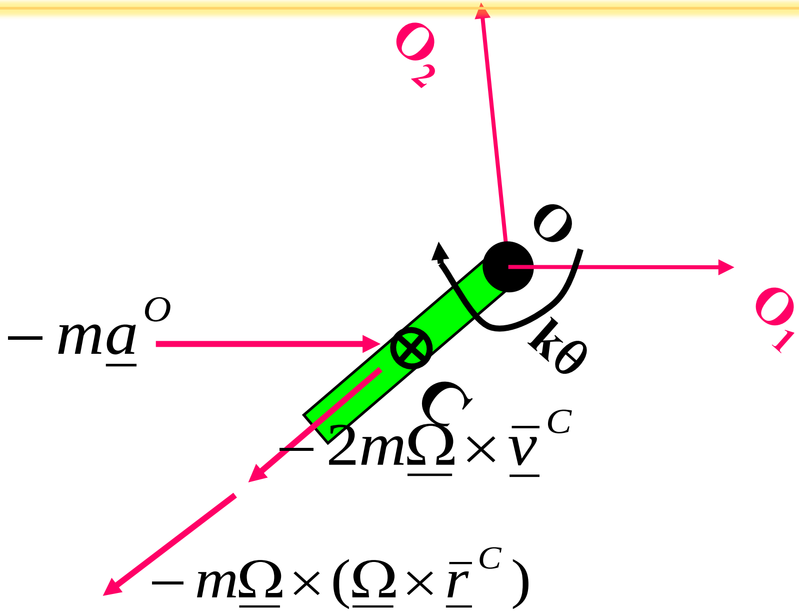
Note that: there exist no coordinate couples,since the reference frame rotates at a constant angular velocity and therigid body is in plane motion, and a central principal axis of Inertia is assumed to be perpendicular to the plane.



2. **Kinetics Analysis**: **Now, "O" is a fixed point in the NNRF, and therefore an admissible *Eulerian Moment Center*. Therefore:**

$$\underline{\bar{M}}^O = \underline{\underline{\bar{I}}}^O \underline{\bar{\alpha}} \Rightarrow -k\theta + (ma\Omega^2)(\frac{\ell}{2}\sin\theta) = \frac{1}{3}m\ell^2\ddot{\theta}$$

$$\ddot{\theta} - \frac{3}{2}\frac{a\Omega^2}{\ell}\sin\theta + \frac{3k}{m\ell^2}\theta = 0 \qquad (D.E.M.)$$

Remarks:

This example should not be constituted to reason that the NNRF is always simpler, but it only shows that the analyst should be free to choose whatever reference frame that best simplifies the analysis and must be specific in stating his frame of reference.

متشکرم

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