

Homework Assignment: # 1

1. By direct expansion show that:

a) $\gamma_{ijk}\gamma_{ijk} = 6$

b) $\gamma_{ijk}d_j d_k = 0$

2. Use the indicial notations method to prove the followings:

a)

$$\underline{A} \times (\underline{B} \times \underline{C}) = (\underline{A} \cdot \underline{C})\underline{B} - (\underline{A} \cdot \underline{B})\underline{C} \quad , \quad \{H \text{ int : } \gamma_{ijk}\gamma_{ipq} = (\delta_{jp}\delta_{kq} - \delta_{jq}\delta_{kp}) \quad \}$$

b)

$$\underline{A} \cdot (\underline{B} \times \underline{C}) = \underline{B} \cdot (\underline{C} \times \underline{A}) = \underline{C} \cdot (\underline{A} \times \underline{B})$$

c)

$$(\underline{A} \times \underline{B}) \cdot (\underline{C} \times \underline{D}) = (\underline{A} \cdot \underline{C})(\underline{B} \cdot \underline{D}) - (\underline{A} \cdot \underline{D})(\underline{B} \cdot \underline{C})$$

d)

$$\nabla \times (\underline{a} \times \underline{b}) = (\underline{b} \cdot \nabla)\underline{a} - \underline{b}(\nabla \cdot \underline{a}) + \underline{a}(\nabla \cdot \underline{b}) - (\underline{a} \cdot \nabla)\underline{b}$$

e)

$$\text{Curl}(\text{Curl } \underline{\mathbf{V}}) = \text{Grad.}(\text{Div. } \underline{\mathbf{V}}) - \nabla^2 \underline{\mathbf{V}} \quad \text{or} \quad \nabla \times (\nabla \times \underline{\mathbf{V}}) = \nabla(\nabla \cdot \underline{\mathbf{V}}) - \nabla^2 \underline{\mathbf{V}}$$

f)

$$\nabla \times \nabla \phi = 0$$

3. Find the Divergence and the Curl of:

$$\underline{F} = (x_1^2 + x_2 x_3) \underline{e}_1 + (x_2^2 + x_1 x_3) \underline{e}_2 + (x_3^2 + x_1 x_2) \underline{e}_3$$