Two Phase Flows
(Section 3)
The Basic Model

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Homework set#1

Problems 1, 2, 3, 4; Chapter 1, Collier and Thome.

Due to next Tuesday (Mehr, 14th)
Time Averaging

\[ \overline{B} = \frac{1}{T} \int_{0}^{T} B \, dt \]
Volume Averaging

\[ \hat{B} = \frac{1}{V} \int_{V} B \, dV \]

\[ \{B\} = \frac{1}{V_c} \int_{V_c} B \, dV \]

Relation between phase and volume averaging for continues phase

\[ \{B\} = \alpha_c \hat{B} \]

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The Basic Model
Principal Equation of Two Phase Flow

1. One dimensional flow
2. Steady state
3. Constant physical properties
4. Existence of Thermodynamic equilibrium
Conservation of Mass

\[
\frac{\partial}{\partial t} (A \alpha_k \rho_k) + \frac{\partial}{\partial Z} (A \alpha_k \rho_k u_k) = \Gamma_k
\]

Continuity equation of phase k

\[
\sum_k \Gamma_k = 0
\]

For steady state flow

\[
\frac{\partial}{\partial t} (A \alpha_k \rho_k) = 0
\]

For gas (g)- liquid (f) two phase flow

\[
\frac{d}{dz} (A_g \rho_g u_g) = \Gamma_g
\]

\[
\frac{d}{dz} (A_f \rho_f u_f) = \Gamma_f
\]

\[
\Gamma_g = -\Gamma_f = \frac{dW_g}{dz} = -\frac{dW_f}{dz}
\]

Void fraction of phase k

Density of phase k

Mass Generation rate per unit length for phase k

\(\alpha_k\)

\(\rho_k\)

\(\Gamma_k\)
Conservation of Momentum

\[
\frac{\partial}{\partial t} (W_k \delta z) + (W_k u_k + \delta z \frac{\partial}{\partial z} (W_k u_k)) - W_k u_k = \frac{\partial}{\partial t} (W_k \delta z) + \delta z \frac{\partial}{\partial z} (W_k u_k)
\]

- **Rate of creation of momentum of phase k**
- **Rate of inflow of momentum within the control volume**

\[
A \alpha_k p - \left( A \alpha_k p + \delta z \frac{\partial}{\partial z} (A \alpha_k p) \right) - \left\{ p \left( -\delta z \frac{\partial}{\partial z} (A \alpha_k) \right) \right\} - A \alpha_k \rho_k \delta z g \sin \theta - \tau_{kw} \rho_k \delta z g \sin \theta - \tau_{kw} P_{kw} \delta z + \sum_{i=1}^{n} \tau_{kz} P_{kn} \delta z + u_k \Gamma_k
\]

- **Gravity**
- **Wall shear**
- **Interfacial shear**
- **Rate of momentum generation**

The Basic Model
Conservation of Momentum

\[ \sum \text{Force} = \text{creation of momentum} + \text{inflow of momentum within the control volume} \]

\[-A \alpha_k \frac{dp}{dz} \delta z - \tau_{kw} P_{kw} \delta z + \sum_{1}^{n} \tau_{knz} P_{kn} \delta z - A \alpha_k \rho_k \delta z \ g \ \sin \theta + u_k \Gamma_k \]

\[= \frac{\partial}{\partial t} (W_k \delta z) + \delta z \frac{\partial}{\partial z} (W_k u_k) \]

Steady gas-liquid two phase flow

\[-A_g dp - \tau_{gw} P_{gw} dz + \tau_{gf} P_{gf} dz - A_g \rho_g dz \ g \ \sin \theta + u_g \Gamma_g = W_g du_g \quad \text{I} \]

\[-A_f dp - \tau_{fw} P_{fw} dz + \tau_{fg} P_{fg} dz - A_f \rho_f dz \ g \ \sin \theta + u_f \Gamma_f = W_f du_f \quad \text{II} \]

Momentum conservation at interface

\[\tau_{gf} P_{gf} dz + u_g \Gamma_g = \tau_{fg} P_{fg} dz + u_f \Gamma_f \quad \text{III} \]

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The Basic Model
Conservation of Momentum

### Summation of Equations I, II, & III

\[-A \frac{dp}{dz} - \tau_{gw} P_{gw} \, dz - \tau_{fw} P_{fw} \, dz - g \sin \theta (\rho_f + A \rho_g) = d (W_g u_g + W_f u_f)\]

*Friction force for each phase*

\[
\begin{align*}
(dF_g + S) &= -\tau_{gw} P_{gw} \, dz - \tau_{gf} P_{gf} \, dz = -A_g \left( \frac{dp}{dz} \right) g \, F \, dz \\
(dF_f - S) &= -\tau_{fw} P_{fw} \, dz + \tau_{gf} P_{gf} \, dz = -A_f \left( \frac{dp}{dz} \right) f \, F \, dz \\
(dF_g + dF_f) &= -\tau_{gw} P_{gw} \, dz - \tau_{fw} P_{fw} \, dz = -A \left( \frac{dp}{dz} \right) F \, dz
\end{align*}
\]

**Part of total pressure gradient which is need for prevalence of friction**
Conservation of Momentum

substitution equation ** in * yields

\[
\frac{dP}{dz} = \frac{dP}{dz} F + \frac{dP}{dz} a + \frac{dP}{dz} z
\]

\[
-\left(\frac{dP}{dz} a\right) = \frac{1}{A} \frac{d}{dz} \left(W_g u_g + W_f u_f\right) = G^2 \frac{d}{dz} \left[\frac{x^2 \nu_g + (1-x)^2 \nu_f}{\alpha (1-\alpha)}\right]
\]

\[
-\left(\frac{dP}{dz} z\right) = g \sin \theta \left[\frac{A_g}{A} \rho_g + \frac{A_f}{A} \rho_f\right] = g \sin \theta \left[\alpha \rho_g + (1-\alpha) \rho_f\right]
\]

Total pressure lost
Energy Conservation

\[ \frac{\partial}{\partial t} \left[ \alpha_k \rho_k (\varepsilon_k + \frac{u_k^2}{2}) A \delta z \right] + W_k (\varepsilon_k + \frac{u_k^2}{2}) \delta z - [W_k (\varepsilon_k + \frac{u_k^2}{2}) - \delta z \frac{\partial}{\partial z} W_k (\varepsilon_k + \frac{u_k^2}{2})] \]

- **rate of increase of total energy in the C.V**
- **rate of entrance of energy within the control volume**

**Internal energy per unit mass**

\[ \varepsilon_k \]

**Rate of heat entrance to C.V of phase k**

\[ \phi_{kw} P_{kw} \delta z + \sum_{1}^{n} \phi_{kn} P_{kn} \delta z + \phi_k A \alpha_k \delta z \]

**Heat flow from channel wall**

**H.V. via the various interfaces**

**Internal heat generation within C.V**

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*The Basic Model*
Energy Conservation

The work done by pressure forces

\[
\left[ \frac{W_k p}{\rho_k} - \left( \frac{W_k p}{\rho_k} + \left( \delta z \frac{\partial}{\partial z} \left( \frac{W_k p}{\rho_k} \right) \right) \right) \right] - W_k g \sin \theta \delta z - pA \frac{\partial \alpha_k}{\partial t}
\]

Work done by expansion of phase k

The work done by body force

Work done by pressure and shear forces at the interface with the other phases

\[
+ \Gamma_k \frac{\delta z p}{\rho_k} + u_k \sum_{1}^{n} \tau_{kn} P_{kn} \delta z
\]

Mass generation rate per unit length

\[
\Gamma_k \delta z \left( \varepsilon_k + \frac{u_k^2}{2} \right)
\]

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Energy Conservation

\[
\frac{\partial}{\partial t} A \alpha_k \rho_k (\varepsilon_k + \frac{u_k^2}{2}) + \frac{\partial}{\partial z} W_k (i_k + \frac{u_k^2}{2}) = -W_k g \sin\theta + \phi_{wk} P_{wk} \\
+ \sum_{1}^{n} \phi_{kn} P_{kn} + \phi_k A \alpha_k - pA \frac{\partial \alpha_k}{\partial t} + \Gamma_k (i_k + \frac{u_k^2}{2}) + u_k \sum_{1}^{n} \tau_{kn} P_{kn}
\]

\[
i_k = u_k + \frac{p}{\rho_k}
\]

For steady gas-liquid two-phase flow in channel with constant area

\[
d \left[ W_g (i_g + \frac{u_g^2}{2}) \right] + W_g g \sin\theta \delta z = \\
\phi_{wg} P_{wg} \delta z + \phi_{gf} P_{gf} \delta z + u_g \tau_{gf} P_{gf} \delta z + \Gamma_g \delta z (i_g + \frac{u_g^2}{2})
\]

\[
d \left[ W_f (i_f + \frac{u_f^2}{2}) \right] + W_f g \sin\theta \delta z = \\
\phi_{wf} P_{wf} \delta z + \phi_{fg} P_{fg} \delta z + u_f \tau_{fg} P_{fg} \delta z + \Gamma_f \delta z (i_f + \frac{u_f^2}{2})
\]

\[†\]
Energy conservation at interface

\[ \phi_{gf} P_{gf} + u_g \tau_{gf} P_{gf} \delta z + \Gamma_g (i_g + \frac{u_g^2}{2}) = \phi_{wf} P_{wf} \delta z + \phi_{fg} P_{fg} \delta z + u_f \tau_{fg} P_{fg} \delta z + \Gamma_f \delta z (i_f + \frac{u_f^2}{2}) \]

with regard the equations †, ‡ and ⁴

\[ \frac{d}{dz} \left[ W_g i_g + W_f i_f \right] + \frac{d}{dz} \left[ \frac{W_g u_g^2}{2} + \frac{W_f u_f^2}{2} \right] + (W_g + W_f) g \sin \theta = Q_{wl} \]

Heat transfer to the fluid across the channel wall per unit length

\[ Q_{wl} = \phi_{wf} P_{wf} + \phi_{wg} P_{wg} \]

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The Basic Model
Energy Conservation

\[ \frac{dp}{dz} [xv_g + (1-x)v_f] = \frac{dE}{dz} - \frac{Q_{wf}}{W} \]

\[ + \left\{ p \frac{d}{dz} [xv_g + (1-x)v_f] + \frac{G^2}{2} \frac{d}{dz} \left[ \frac{x^3v_g^2}{\alpha^2} + \frac{(1-x)^3v_f^2}{(1-\alpha)^2} \right] \right\} + g \sin\theta \]

Total pressure gradient

Frictional dissipation

Acceleration head term

Static head term

Internal energy per unit mass

\[ E = x \varepsilon_g + (1-x)\varepsilon_f \]

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The Basic Model
Use of the momentum or energy equation to evaluate the pressure gradient

Using momentum equation

Using void fraction to calculate acceleration term from

\[-\left(\frac{dP}{dz}\right) a = \frac{1}{A} \frac{d}{dz} (W_g u_g + W_f u_f) = G^2 \frac{d}{dz} \left[ \frac{x^2 v_g}{\alpha} + \frac{(1-x)^2 v_f}{(1-\alpha)} \right] \]

or static head term from

\[-\left(\frac{dP}{dz}\right) z = g \sin\theta \left[ \frac{A_g}{A} \rho_g + \frac{A_f}{A} \rho_f \right] = g \sin\theta \left[ \alpha \rho_g + (1-\alpha) \rho_f \right] \]

Then calculating friction pressure term from correlation equation in terms of independent variables.

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Use of the momentum or energy equation to evaluate the pressure gradient

Using energy equation

• Calculation of pressure lost arising from variation of potential energy
• Calculation of pressure lost arising from variation of kinetic energy
• Calculate the friction pressure term from independent variables

Note: in two methods we need to the void fraction but the degree of importance in each method is not the same.