For simplification, consider an average temperature for hot and cold flows, namely eutropic average temperature.

\[ T_{SA, H} = \frac{Q_H}{\delta Q_H} = -m_H c_H \left( T_{H2} - T_{H1} \right) \Rightarrow T_{SA, H} = \frac{1}{\ln \left( \frac{T_{H2}}{T_{H1}} \right)} \]

2nd Law of thermodynamics:

\[ T_{SA, L} = \frac{(T_{L2} - T_{L1})}{\ln \left( \frac{T_{L2}}{T_{L1}} \right)} \]

\[ \eta = \frac{\delta Q_H - \delta Q_L}{\delta Q_H} = 1 - \frac{T_{SA, L}}{T_{SA, H}} - \frac{T_{SA, L}}{\delta Q_H} \]

\[ S_{gen} = \frac{-\delta Q_H}{T_{SA, H}} + \frac{\delta Q_L}{T_{SA, L}} \Rightarrow S_{gen} = \delta Q_L \left[ \frac{1}{T_{SA, L}} - \frac{1}{T_{SA, H}} \right] \]
Investigation of exergy Eq. terms at different situations:

If a system is in contact with only one source:

\[
\Delta u = q - w \\
\Delta s = \frac{q}{T} + s_{gen} \]  \Rightarrow \Delta u - T_0 \Delta s = q \left(1 - \frac{T_0}{T}\right) - w - T_0 s_{gen}
\]

\[
\Rightarrow \Delta u + P_0 \Delta v - T_0 \Delta s = q \left(1 - \frac{T_0}{T}\right) - (w - P_0 \Delta v) - T_0 s_{gen}
\]

\[
\Delta(u + P_0 v - T_0 s) = \Delta \Phi \quad \text{Exergy or Availability}
\]

\[
\Delta u + P_0 \Delta v - T_0 \Delta s = (u_2 - u_1) + P_0 (v_2 - v_1) - T_0 (s_2 - s_1) = \Phi_2 - \Phi_1
\]

\[
\Rightarrow W_{rev}^{1-2} = \Phi_1 - \Phi_2 = -\Delta \Phi
\]
Net availability convected into the C.V. with mass flow, also called physical exergy, can be split into thermal and pressure components. 

For an ideal gas with constant heat capacity, 

\[ E_{ph.} = c_p \left[ (T - T_0) - T_0 \ln \left( \frac{T}{T_0} \right) \right] + RT_0 \ln \left( \frac{P}{P_0} \right) \]

For solids and liquids \( c_p = c_v = c \)

\[ E_{ph.} = -c (T - T_0) \ln \left( \frac{T}{T_0} \right) + v_m (P - P_0) \]

\( v_m \) is the specific volume determined at \( T_0 \)
The chemical exergy, $E_{x_{\text{chem}}}$ at $P_0$ can be calculated by bringing the pure component in chemical equilibrium with the environment.

\[
E_{x_{\text{chem}}} = RT_0 \ln \left( \frac{P_0}{P_{\text{ref}}} \right)
\]

When a substance does not exist in the environment, it must first react to references substances in order to get in equilibrium with the environment.

The reaction exergy of reference condition needs Gibbs free energy changes.

\[
\Delta G^0 = \Delta H^0 - T_0 \Delta S^0
\]

\[
E_{x_{\text{chem}}} = \sum_{i=1}^{n} v_i E_{x_{\text{chem},\text{ref}}} - \Delta G^0
\]
Exergy analysis (2nd law analysis), 2nd law efficiency

Heat Engines: $\eta_{2nd\ law} = \frac{W_{\text{actual}}}{W_{\text{reversible}}}$

Non-Cyclic Process: $\eta_{2nd\ law} = \frac{W_{\text{actual}}}{-\Delta\Phi} = \frac{W_{\text{actual}}}{W_{\text{rev}}}_{1-2}$

Energy analysis (1st law analysis), 1st law efficiency

Heat Engines: $\eta_{th} = \frac{W}{Q_H}$

Non-Cyclic Process: $\eta_{is} = \frac{W_{\text{actual}}}{W_{\text{isentropic}}}$