# Two Phase Flows

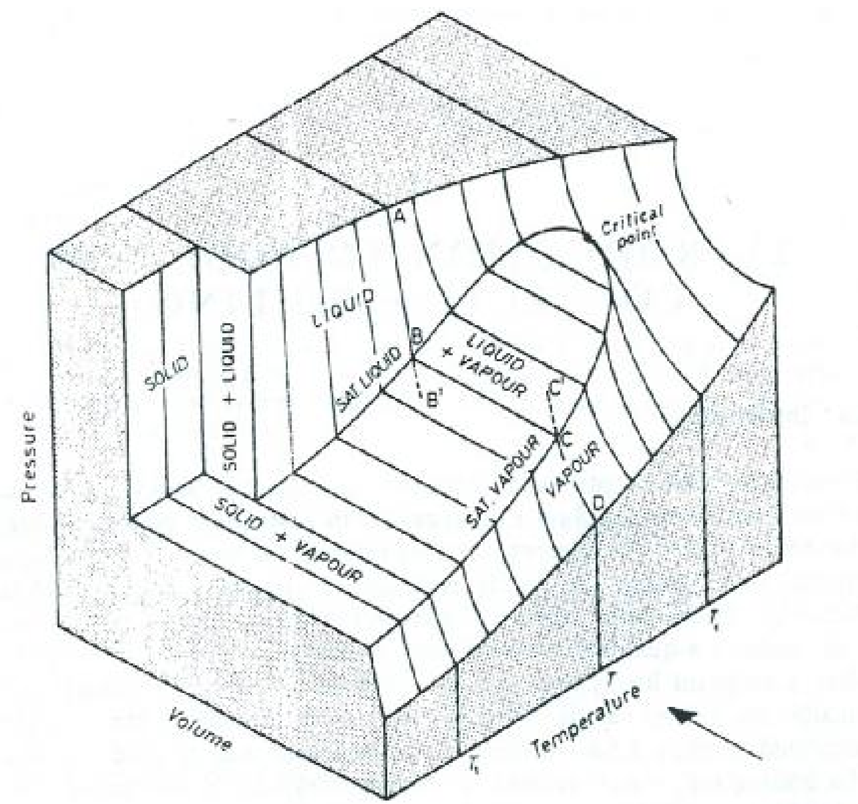
(Section 9)

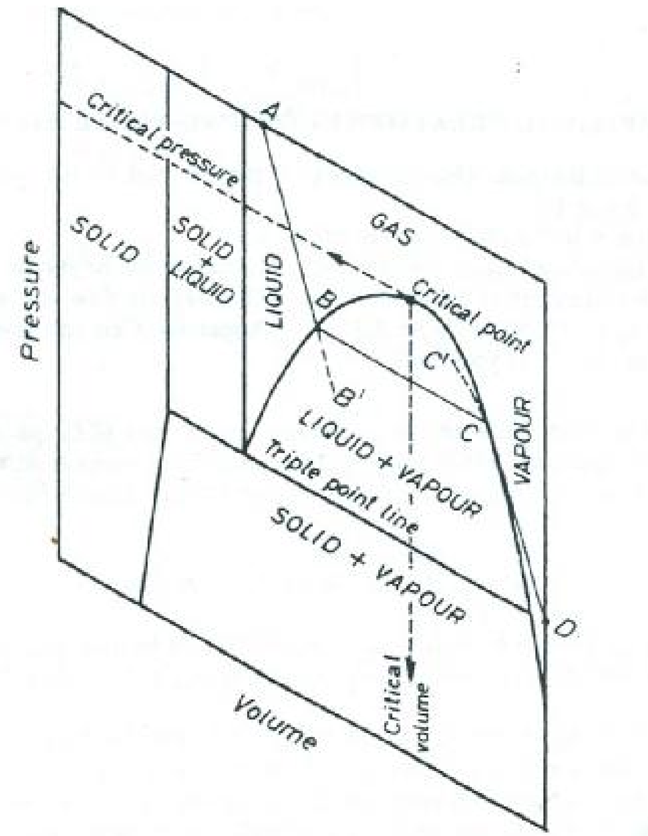
Introduction to Pool and Convective Boiling

By: Prof. M. H. Saidi

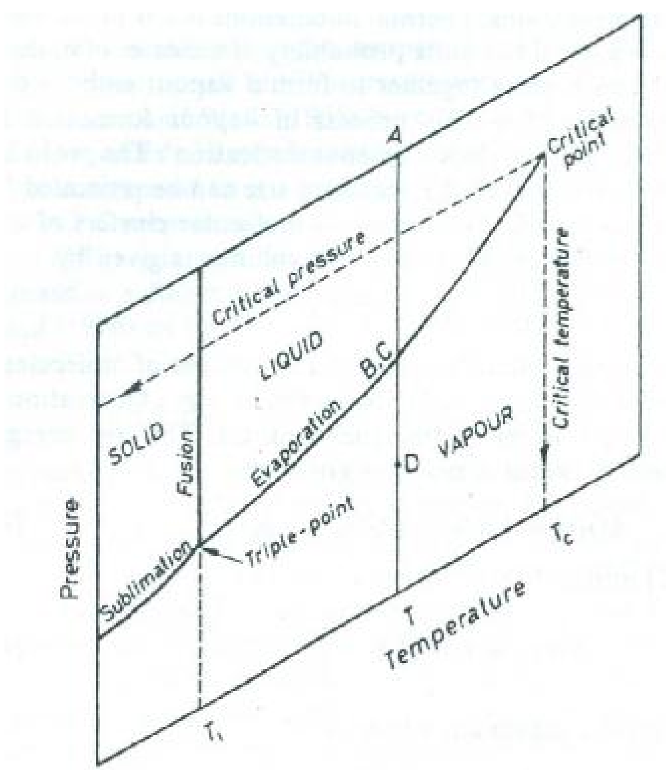
Center of Excellence in Energy Conversion
School of Mechanical Engineering
Sharif University of Technology

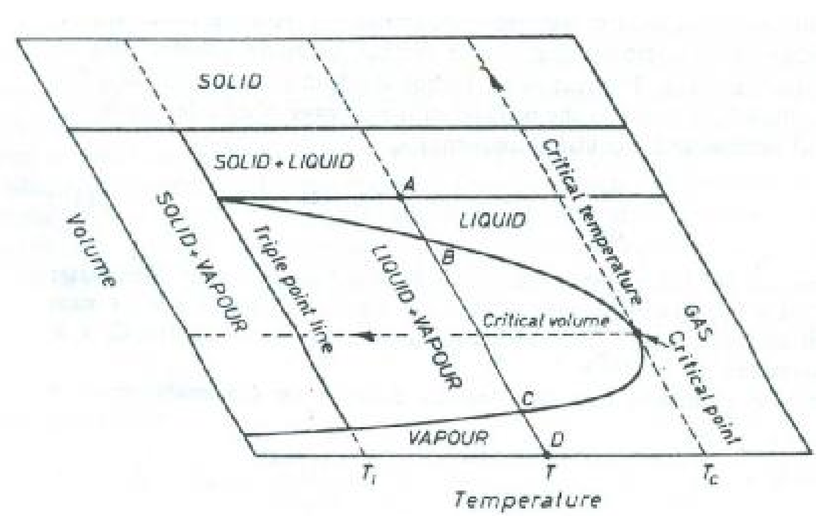
# Elementary Thermodynamics of Vapor/Liquid Systems



# Elementary Thermodynamics of Vapor/Liquid Systems



# Basic Process of Boiling

Superheat Requirements for vapour nucleation:

$$p_g - p_f = \frac{2s}{r^*}$$

$$p_g = p_\infty \exp\left(-2s\, v_f M / r^* RT\right) \approx p_\infty \left(1 - \frac{2s\, v_f}{p_\infty r^* v_g}\right)$$

$$p_\infty - p_g = \frac{2s}{r^*}\left(1 + \frac{v_f}{v_g}\right)$$

Using Clausius-Clapeyron equation:

$$dp \big/ dT = \frac{Ji_{fg}}{T\left[v_g - v_f\right]}$$

With this assumptions:

$$v_g \quad v_f \; and \; Mpv_g = RT$$

$$\frac{1}{p}dp = \frac{Ji_{fg}M}{RT^2}dT$$

# Basic Process of Boiling

Integrating previous equation:

$$\ln\left(\frac{p_\infty}{p_f}\right) = -\frac{Ji_{fg}M}{R}\left(\frac{1}{T_g} - \frac{1}{T_{SAT}}\right) = \frac{Ji_{fg}M}{RT_gT_{SAT}}\left(T_g - T_{SAT}\right)$$

$$p_\infty - p_g = \frac{2s}{r^*}\left(1 + \frac{v_f}{v_g}\right)$$

$$\left(T_g - T_{SAT}\right) = \frac{RT_gT_{SAT}}{Ji_{fg}M}\ln\left[1 + \frac{2s}{p_f r^*}\left(1 + \frac{v_f}{v_g}\right)\right]$$

With this assumptions:

$$v_g \qquad v_f \;\; and \;\; \frac{2s}{p_f r^*} \qquad 1$$

$$\left(T_g - T_{SAT}\right) = \Delta T_{SAT} = \frac{RT_{SAT}^2}{Ji_{fg}M}\frac{2s}{p_f r^*}$$

$$l = kT_g/h = (2s/pm)^{1/2}$$

# Homogeneous nucleation

Boltzmann Equation: $N(r) = Ne^{-\Delta G(r)/kT_g}$

$$\Delta G(r) = 4p r^2 s - \frac{4}{3} p r^3 (p_g - p_f)$$

Using:

$$p_g - p_f = \frac{2s}{r^*}$$

$$\Delta G(r) = 4p r^2 s \left(1 - \frac{2r}{3r^*}\right)$$

$$\max(\Delta G) = \Delta G(r^*) = \frac{4}{3} p r^2 s = \frac{16}{3} \frac{p s^3}{(p_g - p_f)^2}$$

Rate of nucleation: $\frac{dn}{dt} = l N(r^*) = l Ne^{-\Delta G(r^*)/kT_g}$

Lienhard equation: $T_{rg} - T_{rSAT} = 0.905 - T_{rSAT} + 0.095 T_{rSAT}^8$

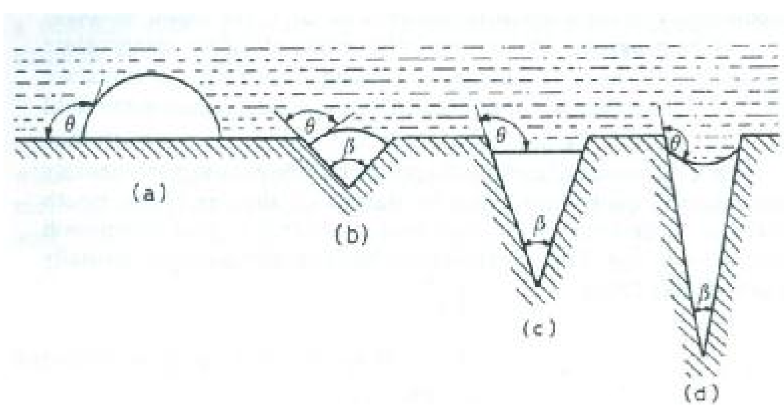
# Heterogeneous Nucleation

$$p_g + p_a - p_f = \frac{2s}{r^*}$$

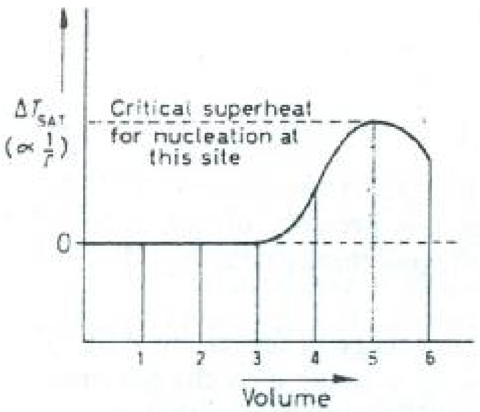
$$\left(T_g - T_{SAT}\right) = \frac{RT_g T_{SAT}}{Ji_{fg} M} \ln\left[1 + \frac{2s}{p_f r^*}\left(1 + \frac{v_f}{v_g}\right) - \frac{p_a}{p_f}\right]$$

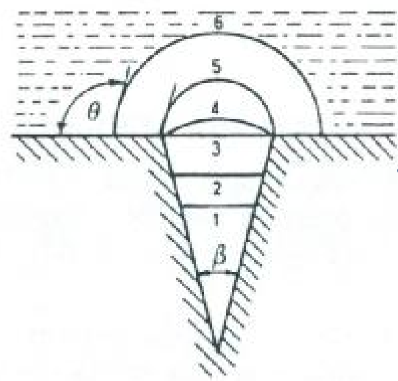
Nucleation at solid surfaces:

$$f = \frac{2 + 2\cos q + \cos q \sin^2 q}{4}$$



# Heterogeneous Nucleation




Nucleation from cavities

Formation of an active site

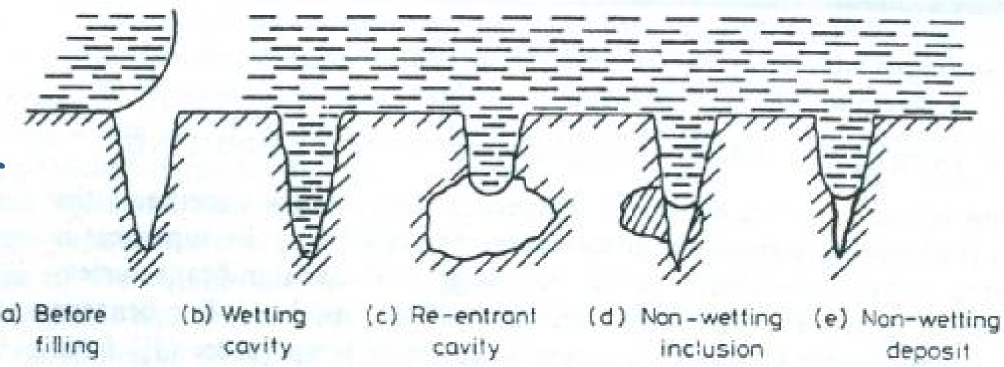
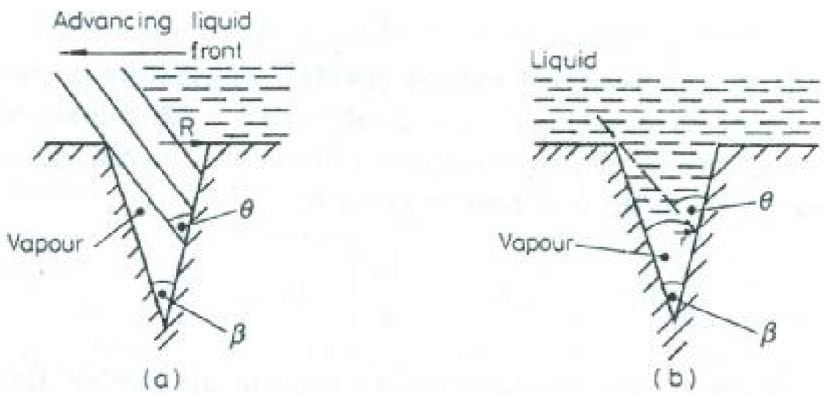
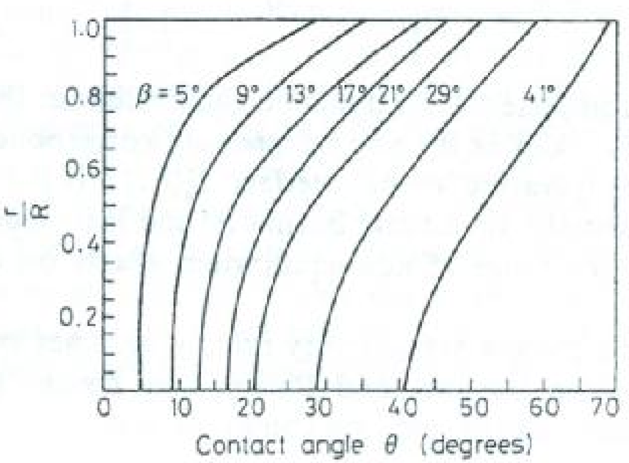



Fig. 4.7. Formation of an active site.

# Heterogeneous Nucleation




Vapour trapping model of Lorentz et al. (1974), for sizing active cavities

# Simple bubble dynamic

$$\frac{1}{2} r_f \int_R^\infty 4p r^2 \dot{r}^2 dr = \frac{4}{3}p\left(R^3 - R_0^3\right)\Delta p$$

Mechanical energy Eq. *

Assumptions:
- Spherical element
- Non-viscous fluid
- Incompressible fluid

$$\dot{r} = \left(\frac{R}{r}\right)^2 \dot{R}$$

Substituting into * yields:

$$2p r_f R^3 \dot{R}^2 = \frac{4}{3}p\left(R^3 - R_0^3\right)\Delta p$$

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{\Delta p}{r_f}$$

Rayleigh Equation

$$\Delta p = p_g(R,t) - \frac{2s}{r} - p_f$$

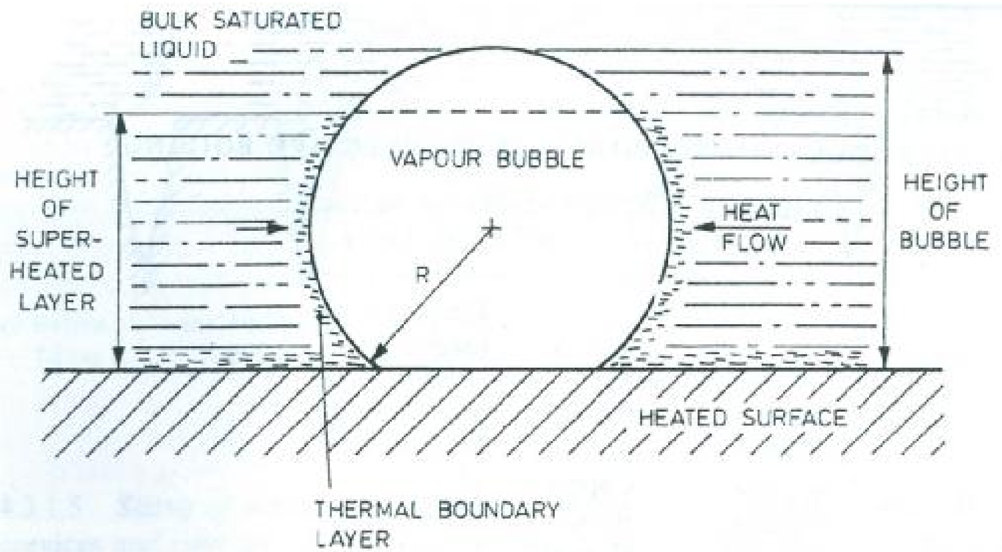
$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = -v_f\left[\frac{2s}{R} + p_f - p_g(R,t)\right]$$

# Simple bubble dynamic

Plesset and Zwick solution:

$$\dot{R} = \left[T_g - T_g(R,t)\right]\left(\frac{k_f}{i_{fg}r_g}\right)\left(\frac{p}{3}a_f t\right)^{-1/2}$$

$$R = \frac{2\Delta T_{SAT} k_f}{i_{fg} r_g}\left(\frac{3t}{p a_f}\right)^{1/2}$$




Mikie et al. (1970)

$$R^+ = \frac{2}{3}\left(\left(t^+ + 1\right)^{3/2} - \left(t^+\right)^{3/2} - 1\right)$$

$$R^+ = RA/B^2,\ t^+ = tA^2/B^2$$

$$A = \left[b\frac{\Delta T_{SAT} i_{fg} r_g}{T_{SAT} r_f}\right]^{1/2}$$

$$B = \left[\frac{12}{p} Ja^2 a_f\right]^{1/2}$$

# Simple bubble dynamic

Van Stralen Model (1970)

$$R=\left[\frac{12}{p}a_f\right]^{1/2} b\left(\frac{r_f c_{pf}}{i_{fg} r_g}\right)\left[\Delta T_{SAT}\exp\left(-t/t_g\right)^{1/2}\right]t^{1/2}$$

Bubble detachment and frequency:

$$D_d=0.0208q\left[s\Big/g\left(r_f-r_g\right)\right]^{1/2}$$

$$fD_d=0.59\left[\frac{s\,g\left(r_f-r_g\right)}{r_f^2}\right]^{1/4}$$