



Two Phase Flows

(Section 5)

The Basic Model

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Homogeneous Model



Assumptions:

- ∅ Velocity of gas and liquid phases are equal. $U_g = U_f$
- ∅ Establishment of thermodynamic equilibrium between phases.
- ∅ Using the single phase friction coefficient which is appropriately defined for two phase flow.

This model appropriate for bubbly and wispy annular regime.

Steady homogeneous

$$W = A \bar{r} \bar{u}$$

continuity

$$-A dp - d\bar{F} - A \bar{r} g dz \sin q = W d\bar{u}$$

momentum

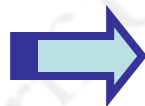
$$dq - dW = di + d\left(\frac{\bar{u}^2}{2}\right) + g \sin q dz$$

energy

$$\bar{n} = \frac{Q}{W} = [x n_g + (1-x) n_f] = [n_f + x n_{fg}] = \frac{j}{G} = \frac{1}{\bar{r}}$$

Specific
volume

$$u_f = u_g = \bar{u}$$



$$\bar{u} = G \bar{n} = j$$

Homogeneous Model

$$a = \frac{x n_g}{\bar{v}} = b \quad , \quad (1-a) = \frac{(1-x) n_f}{\bar{v}} = (1-b)$$

$$d\bar{F} = t_w pdz$$

$$t_w = f_{TP} \left(\frac{\bar{r} \bar{u}^2}{2} \right)$$

Wall shear stress

$$-\left(\frac{dp}{dz} F \right) = \frac{1}{A} \frac{d\bar{F}}{dz} = \frac{t_w P}{A} = \frac{f_{TP} P}{A} \left(\frac{\bar{r} \bar{u}^2}{2} \right)$$

$$\frac{p}{A} = \frac{4}{D}$$

for circular channel

$$-\left(\frac{dp}{dz} F \right) = \frac{2f_{TP} G^2 \bar{n}}{D} = \frac{2f_{TP} Gj}{D}$$

I

Homogeneous Model

$$-\left(\frac{dp}{dz} a\right) = G \frac{d(\bar{u})}{dz} = G^2 \frac{d\bar{n}}{dz} \quad \text{II}$$

$$\frac{d\bar{n}}{dz} = n_{fg} \frac{dx}{dz} + x \frac{dn_g}{dp} \frac{dp}{dz}$$

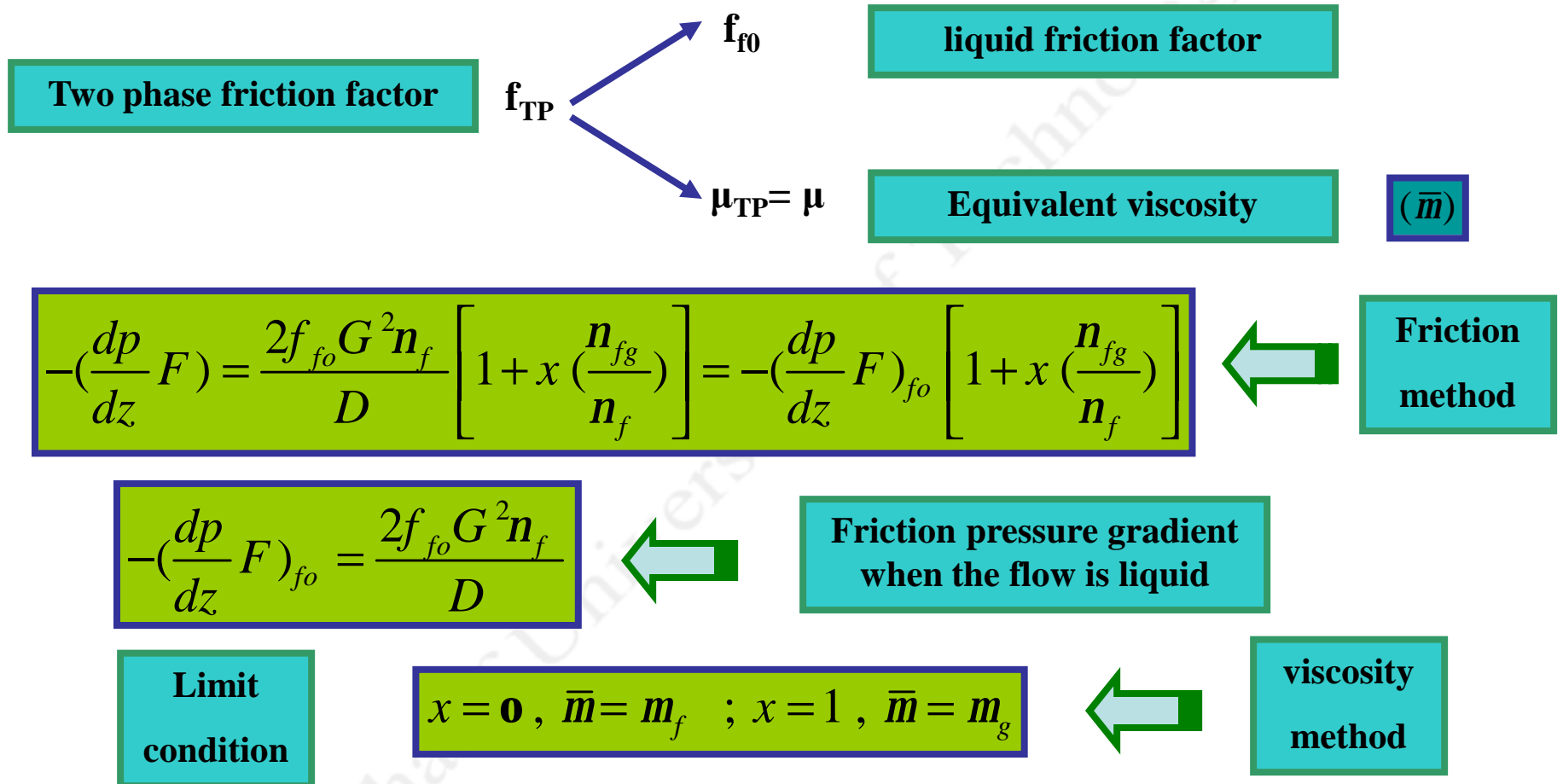
$$-\left(\frac{dp}{dz} z\right) = \bar{r} g \sin q = \frac{g \sin q}{\bar{n}} \quad \text{III}$$

From equations **I**, **II** and **III**



$$-\left(\frac{dp}{dz}\right) = \frac{\frac{2f_{TP} G^2 n_f}{D} \left(1 + x \left(\frac{n_{fg}}{n_f}\right)\right) + G^2 n_f \left(\frac{n_{fg}}{n_f}\right) \frac{dx}{dz} + \frac{g \sin q}{n_f \left(1 + x \left(\frac{n_{fg}}{n_f}\right)\right)}}{1 + G^2 x \left(\frac{dn_g}{dp}\right)}$$

Two-Phase Friction Factor



Viscosity method

$$i) \frac{1}{m} = \frac{x}{m_g} + \frac{(1-x)}{m_f}$$

McAdams et al. (1942)

$$ii) \bar{m} = xm_g + (1-x)m_f$$

Cicchitti et al. (1960)

$$iii) \bar{m} = \bar{r} \left[xn_g m_g + (1-x)n_f m_f \right]$$

Dukler et al. (1964)

$$f_{TP} = 0.079 \left[\frac{GD}{\bar{m}} \right]^{-1/4}$$

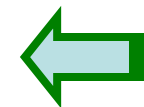
Blasius

$$-\left(\frac{dp}{dz} F\right) = -\left(\frac{dp}{dz} F\right)_{fo} \left[1 + x \left(\frac{n_{fg}}{n_f}\right) \right] \left[1 + x \left(\frac{m_{fg}}{m_g}\right) \right]^{-1/4}$$

$$-\left(\frac{dp}{dz} F\right) = \left(-\frac{dp}{dz} F\right)_{fo} f_{fo}^2$$

$$f_{fo}^2 = \left[1 + x \left(\frac{n_{fg}}{n_f}\right) \right] \left[1 + x \left(\frac{m_{fg}}{m_g}\right) \right]^{-1/4}$$

Two phase
frictional
multiplier



The Basic Model

Two phase frictional multiplier

$$\phi_{fo}^2 = \left[1 + x \left(\frac{v_{fg}}{v_f} \right) \right] \left[1 + x \left(\frac{\mu_{fg}}{\mu_g} \right) \right]^{-1/4}$$

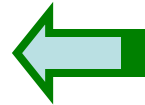
Steam quality % by wt.	Pressure, bar (psia)								
	1.01 (14.7)	6.89 (100)	34.4 (500)	68.9 (1000)	103 (1500)	138 (2000)	172 (2500)	207 (3000)	221.2 (3206)
1	16.21	3.40	1.44	1.19	1.10	1.05	1.04	1.01	1.0
5	67.6	12.18	3.12	1.89	1.49	1.28	1.16	1.06	1.0
10	121.2	21.8	5.06	2.73	1.95	1.56	1.30	1.13	1.0
20	212.2	38.7	7.8	4.27	2.81	2.08	1.60	1.25	1.0
30	292.8	53.5	11.74	5.71	3.60	2.57	1.87	1.36	1.0
40	366	67.3	14.7	7.03	4.36	3.04	2.14	1.48	1.0
50	435	80.2	17.45	8.30	5.08	3.48	2.41	1.60	1.0
60	500	92.4	20.14	9.50	5.76	3.91	2.67	1.71	1.0
70	563	104.2	22.7	10.70	6.44	4.33	2.89	1.82	1.0
80	623	115.7	25.1	11.81	7.08	4.74	3.14	1.93	1.0
90	682	127	27.5	12.90	7.75	5.21	3.37	2.04	1.0
100	738	137.4	29.8	13.98	8.32	5.52	3.60	2.14	1.0



Using model to calculate pressure drop



$$\left| G^2 x \left(\frac{du_g}{dP} \right) \right| \ll 1$$



If compressibility of gas phase be negligible

$$\frac{n_{fg}}{n_f} \text{ and } f_{TP} \text{ are Constant}$$

$$\frac{dx}{dz} = Const.$$

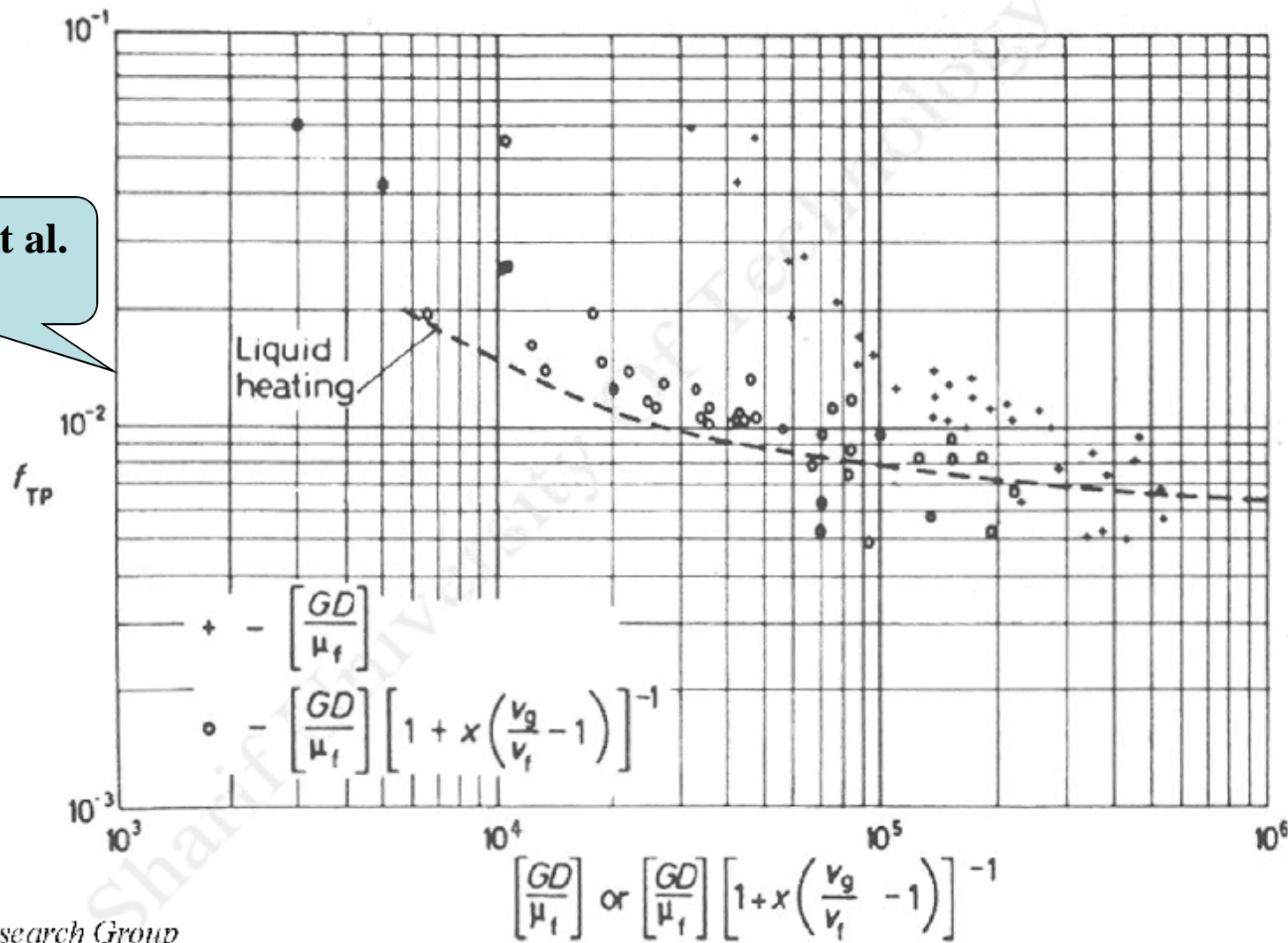
Integrating from state of saturated liquid (x=0) with linear variation of x, yields:

$$\Delta p = \frac{2F_{TP} LG^2 n_f}{D} \left[1 + \frac{x}{2} \left(\frac{n_{fg}}{n_f} \right) \right] + G^2 n_f \left(\frac{n_{fg}}{n_f} \right) x + \frac{gL \sin q}{x n_{fg}} \ln \left[1 + x \left(\frac{n_{fg}}{n_f} \right) \right]$$

Using empirical correlation to calculating friction coefficient in homogenous model



**Davidson et al.
1943**



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