Two Phase Flows

(Section 5)

The Basic Model

By: Prof. M. H. Saidi

Center of Excellence in Energy Conversion

School of Mechanical Engineering

Sharif University of Technology

Homogeneous Model

Assumptions:

- Velocity of gas and liquid phases are equal. $U_g=U_f$
- Establishment of thermodynamic equilibrium between phases.
- Using the single phase friction coefficient which is appropriately defined for two phase flow.

This model appropriate for bubbly and wispy annular regime.

Steady homogeneous

$$W = A\bar{\rho}\bar{u}$$

continuity

$$-A\,dp - d\bar{F} - A\bar{\rho}\,g\,dz\,\sin\theta = W\,d\bar{u}$$

momentum

$$\delta q - \delta W = di + d\left(\frac{\bar{u}^2}{2}\right) + g\sin\theta\,dz$$

energy

$$\bar{\nu} = \frac{Q}{W} = \left[x\nu_g + (1-x)\nu_f\right] = \left[\nu_f + x\nu_{fg}\right] = \frac{j}{G} = \frac{1}{\bar{\rho}}$$

Specific volume

$$u_f = u_g = \bar{u} \quad \Rightarrow \quad \bar{u} = G\bar{\nu} = j$$

Homogeneous Model

$$a = \frac{x n_g}{\bar{v}} = b \quad , \qquad (1-a) = \frac{(1-x) n_f}{\bar{v}} = (1-b)$$

$$d\bar{F} = t_w\, p\, dz$$

$$t_W = f_{Tp}\left(\frac{\bar{r}\,\bar{u}^2}{2}\right)$$

Wall sear stress

$$-\left(\frac{dp}{dz}F\right) = \frac{1}{A}\frac{d\bar{F}}{dz} = \frac{t_w P}{A} = \frac{f_{TP} P}{A}\left(\frac{\bar{r}\,\bar{u}^2}{2}\right)$$

$$\frac{p}{A} = \frac{4}{D}$$

for circular channel

$$-\left(\frac{dp}{dz}F\right) = \frac{2 f_{TP} G^2 \bar{n}}{D} = \frac{2 f_{TP} G j}{D}$$

I

Homogeneous Model

$$\boxed{-(\frac{dp}{dz}a) = G\frac{d(\bar{u})}{dz} = G^2\frac{d\bar{n}}{dz}}$$ **II**

$$\boxed{\frac{d\bar{n}}{dz} = n_{fg}\frac{dx}{dz} + x\frac{dn_g}{dp}\frac{dp}{dz}}$$

$$\boxed{-(\frac{dp}{dz}z) = \bar{r}g\sin q = \frac{g\sin q}{\bar{n}}}$$ **III**

From equations I, II and III

$$\boxed{-(\frac{dp}{dz}) = \frac{\frac{2f_{TP}G^2n_f}{D}(1+x(\frac{n_{fg}}{n_f})) + G^2n_f(\frac{n_{fg}}{n_f})\frac{dx}{dz} + \frac{g\sin q}{n_f(1+x(n_{fg}/n_f))}}{1+G^2x(\frac{dn_g}{dp})}}$$

Two- Phase Friction Factor

Two phase friction factor — f_{TP} → f_{f0} — **liquid friction factor**

f_{TP} → $\mu_{TP} = \mu$ — **Equivalent viscosity** $(\bar{m})$

$$-\left(\frac{dp}{dz}F\right) = \frac{2f_{fo}G^2 n_f}{D}\left[1 + x\left(\frac{n_{fg}}{n_f}\right)\right] = -\left(\frac{dp}{dz}F\right)_{fo}\left[1 + x\left(\frac{n_{fg}}{n_f}\right)\right]$$

Friction method

$$-\left(\frac{dp}{dz}F\right)_{fo} = \frac{2f_{fo}G^2 n_f}{D}$$

Friction pressure gradient when the flow is liquid

Limit condition

$$x = 0\ ,\ \bar{m} = m_f \quad ;\ x = 1\ ,\ \bar{m} = m_g$$

viscosity method

Multiphase Flow Research Group
School of Mechanical Engineering

The Basic Model

Viscosity method

$$i)\ \frac{1}{\bar{m}} = \frac{x}{m_g} + \frac{(1-x)}{m_f}$$

McAdams et al. (1942)

$$ii)\ \bar{m} = x m_g + (1-x) m_f$$

Cicchitti et al. (1960)

$$iii)\ \bar{m} = \bar{r}\left[x n_g m_g + (1-x) n_f m_f\right]$$

Dukler et al. (1964)

$$f_{TP} = 0.079\left[GD \big/ \bar{m}\right]^{-\frac{1}{4}}$$

Blasius

$$-\left(\frac{dp}{dz}F\right) = -\left(\frac{dp}{dz}F\right)_{fo}\left[1 + x\left(\frac{n_{fg}}{n_f}\right)\right]\left[1 + x\left(\frac{m_{fg}}{m_g}\right)\right]^{-1/4}$$

$$-\left(\frac{dp}{dz}F\right) = \left(-\frac{dp}{dz}F\right)_{fo} f_{fo}^{2}$$

$$f_{fo}^{2} = \left[1 + x\left(\frac{n_{fg}}{n_f}\right)\right]\left[1 + x\left(\frac{m_{fg}}{m_g}\right)\right]^{-1/4}$$

Two phase frictional multiplier

The Basic Model

Two phase frictional multiplier

$$\phi_{fo}^2 = \left[1 + x\left(\frac{v_{fg}}{v_f}\right)\right]\left[1 + x\left(\frac{\mu_{fg}}{\mu_g}\right)\right]^{-1/4}$$

Steam quality % by wt.	Pressure, bar (psia)								
	1.01 (14.7)	6.89 (100)	34.4 (500)	68.9 (1000)	103 (1500)	138 (2000)	172 (2500)	207 (3000)	221.2 (3206)
1	16.21	3.40	1.44	1.19	1.10	1.05	1.04	1.01	1.0
5	67.6	12.18	3.12	1.89	1.49	1.28	1.16	1.06	1.0
10	121.2	21.8	5.06	2.73	1.95	1.56	1.30	1.13	1.0
20	212.2	38.7	7.8	4.27	2.81	2.08	1.60	1.25	1.0
30	292.8	53.5	11.74	5.71	3.60	2.57	1.87	1.36	1.0
40	366	67.3	14.7	7.03	4.36	3.04	2.14	1.48	1.0
50	435	80.2	17.45	8.30	5.08	3.48	2.41	1.60	1.0
60	500	92.4	20.14	9.50	5.76	3.91	2.67	1.71	1.0
70	563	104.2	22.7	10.70	6.44	4.33	2.89	1.82	1.0
80	623	115.7	25.1	11.81	7.08	4.74	3.14	1.93	1.0
90	682	127	27.5	12.90	7.75	5.21	3.37	2.04	1.0
100	738	137.4	29.8	13.98	8.32	5.52	3.60	2.14	1.0

Using model to calculate pressure drop

$$\left| G^2 x \left(\frac{du_g}{dP} \right) \right| << 1$$

If compressibility of gas phase be negligible

$$\frac{\nu_{fg}}{\nu_f} \ \textit{and} \ f_{TP} \ \text{are Constant}$$

Integrating from state of saturated liquid (x=0) with linear variation of x, yields:

$$\frac{dx}{dz} = Const.$$

$$\Delta p = \frac{2F_{TP} L G^2 \nu_f}{D} \left[1 + \frac{x}{2} \left(\frac{\nu_{fg}}{\nu_f} \right) \right] + G^2 \nu_f \left(\frac{\nu_{fg}}{\nu_f} \right) x + \frac{gL \sin\theta}{x \nu_{fg}} \ln \left[1 + x \left(\frac{\nu_{fg}}{\nu_f} \right) \right]$$

Using empirical correlation to calculating friction coefficient in homogenous model

Davidson et al. 1943