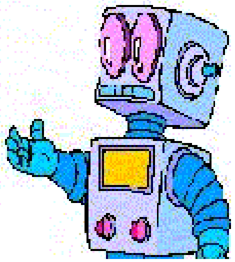
# INTRODUCTION TO ROBOTICS

(Kinematics, Dynamics, and Design)

## SESSION # 10:

## MANIPULATOR KINEMATICS



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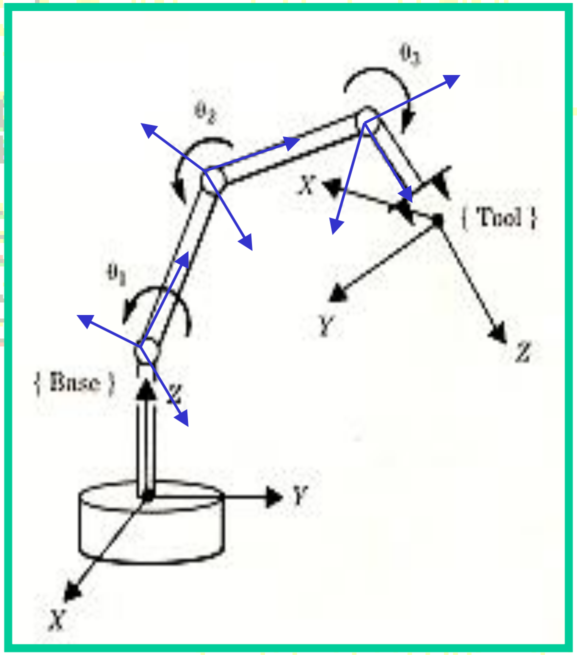
# Manipulator Kinematics

- Study of motion without regard to forces causing the motion (Position, Velocity, and Acceleration).

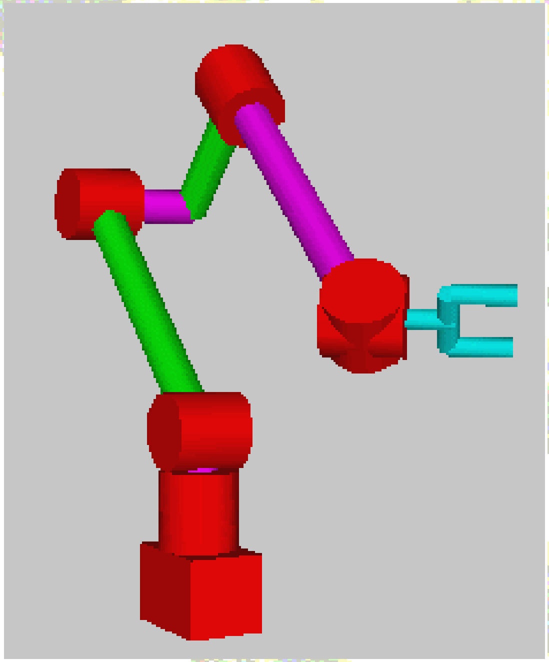
➢ We will describe a method to compute the position and orientation of the manipulator linkages and end-effector as a function of joint variables relative to the base frame.

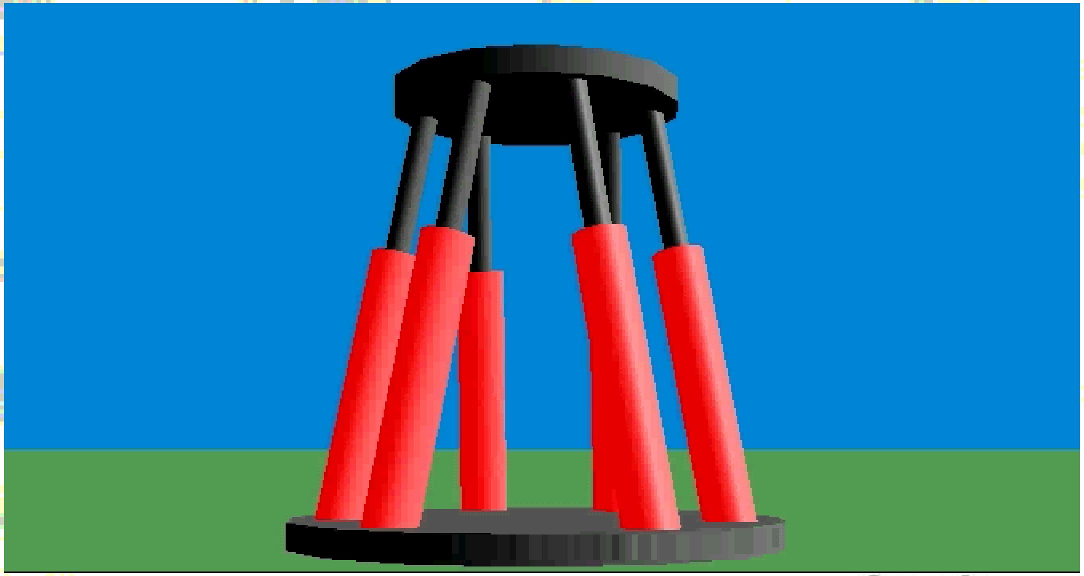
➢ To perform this task, we will affix frames to the various parts of the robot mechanism, and then describe the relationship between these frames.

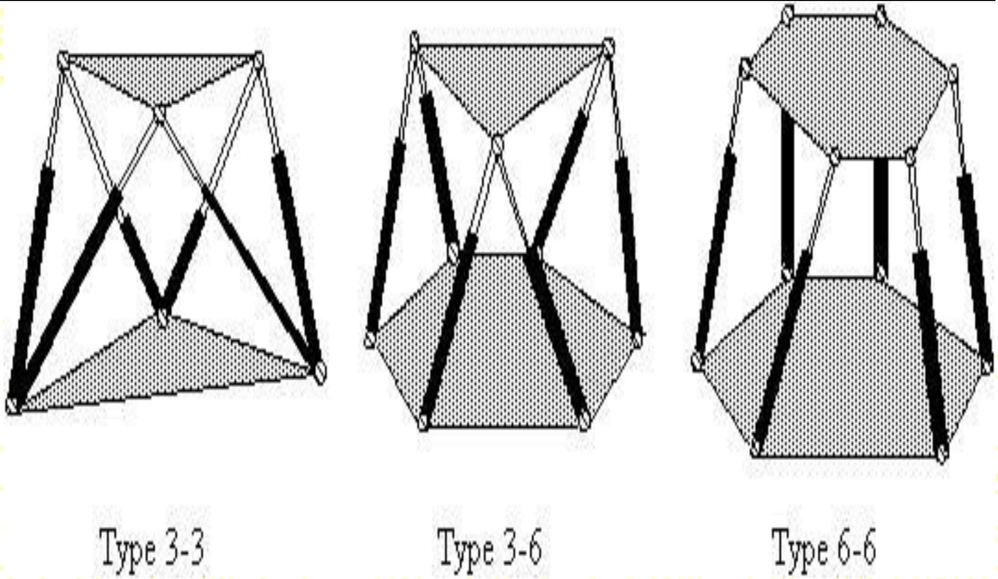



A 3-DOF Manipulator Arm

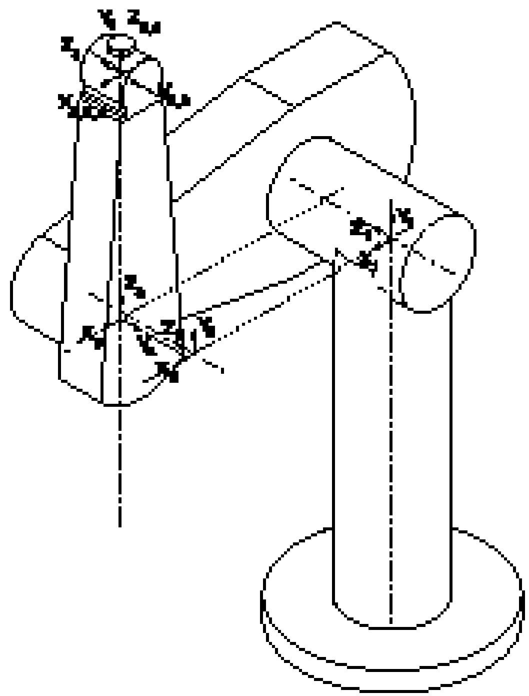
# Serial and Parallel Manipulators







# Serial and Parallel Manipulators

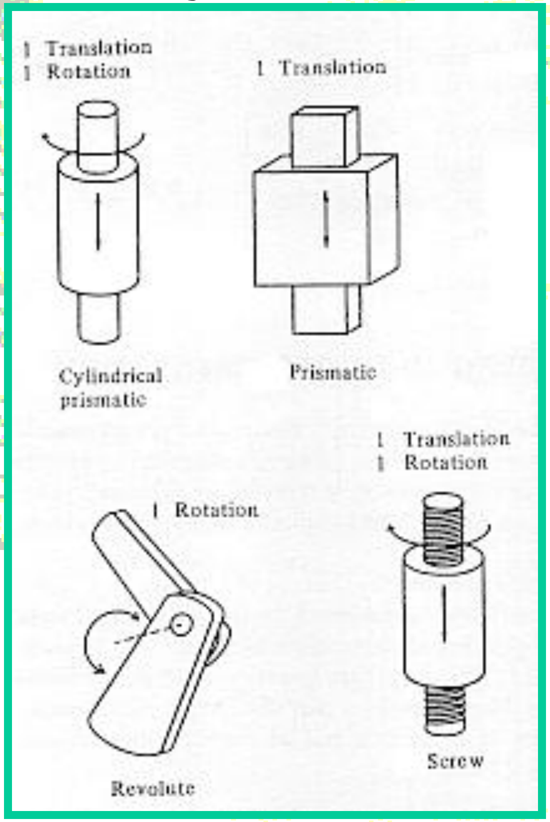


PUMA560

Hexapod

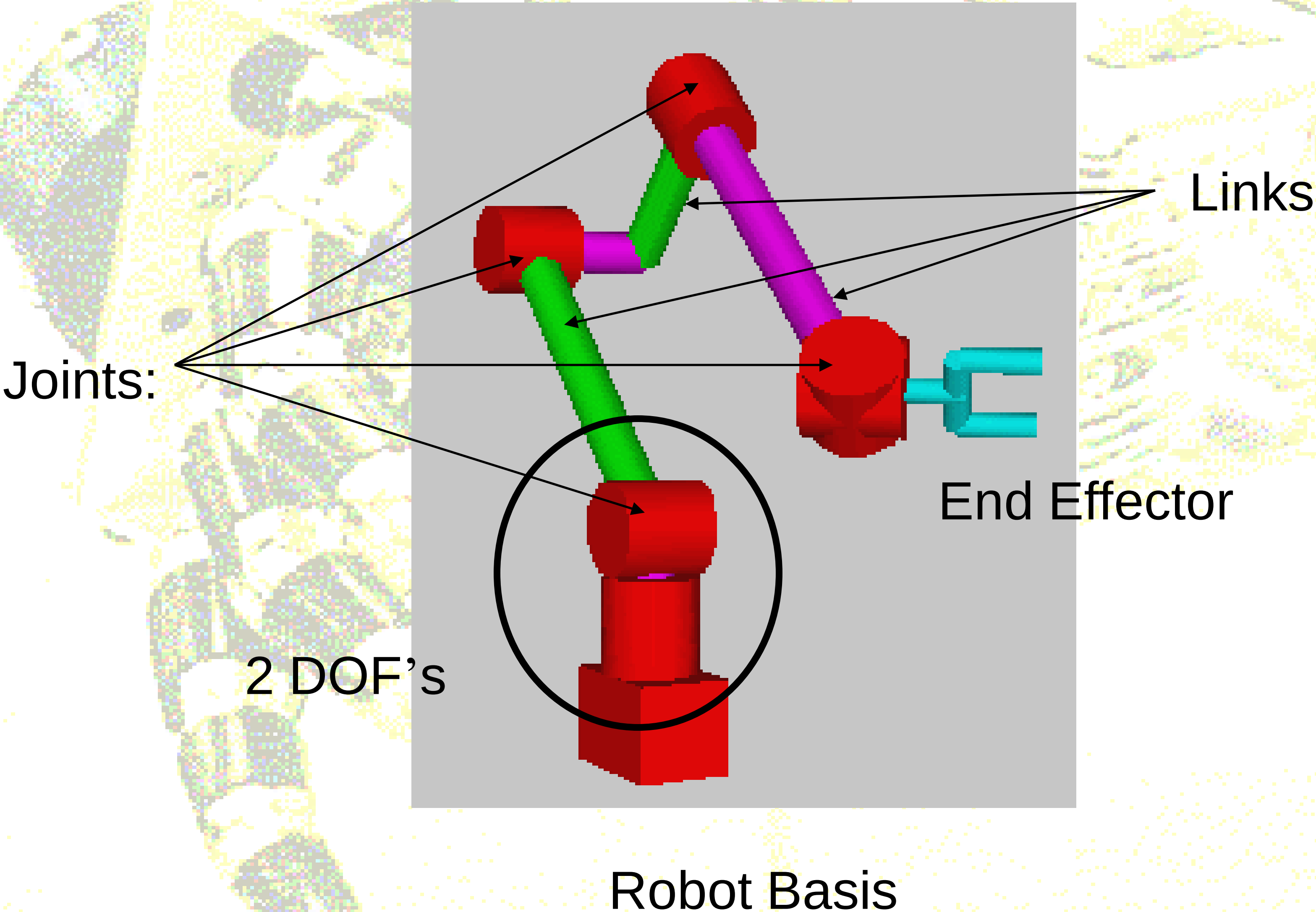
# Manipulator Kinematics

- **Manipulator:** **A set of bodies (links) connected by a series of joints (Revolute/Prismatic)**




- **Joints connect parts of manipulators.**
- **The most common joint types are:**
  - **Revolute joint (rotation around a fixed axis)**
  - **Prismatic joint (linear movement)**
- **These joints provide the DOF for an end-effector.**

# Links and Joints

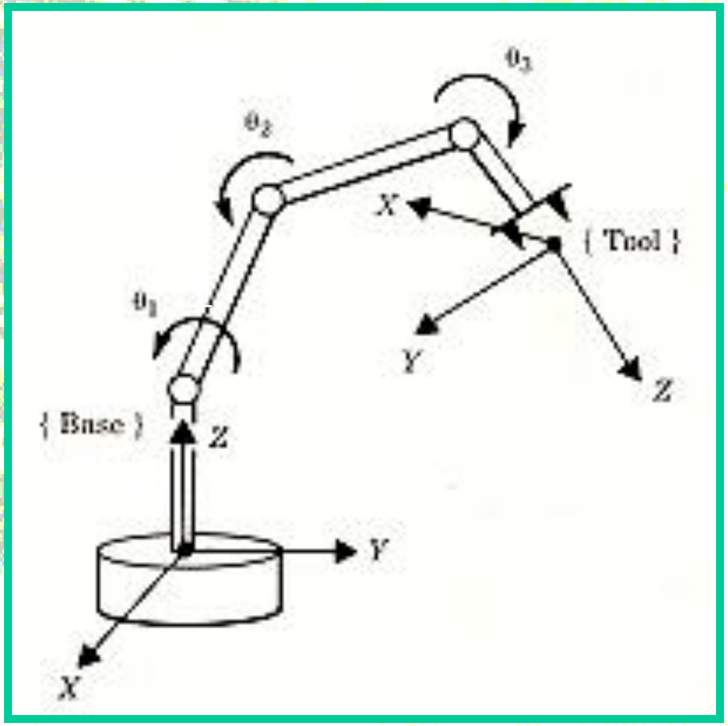


# Manipulator Kinematics

- **Link Description (توصیف یک رابط):**

➢ **Links Numbering Convention:**

| | |
|---|---|
| **Base of the arm:** | **Link-0** |
| **1st moving link:** | **Link-1** |
| **.** | **.** |
| **.** | **.** |
| **.** | **.** |
| **Last moving link** | **Link-n** |




A 3-DOF Manipulator Arm

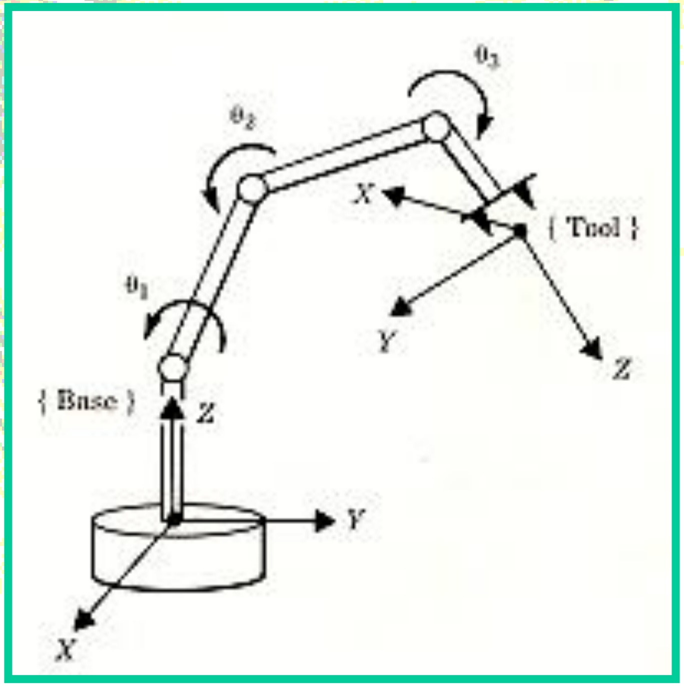
# Degrees-of-Freedom of a Manipulator

- To position an End-Effector (Gripper) generally in 3D-Space, a minimum of 6-joints are required (3 for position & 3 for orientation).

- Typical Manipulators have 5 or 6 joints.

- In open kinematics chains (i.e. Industrial Manipulators):

{# of D.O.F. = # of Joints}




A 3-DOF Manipulator Arm

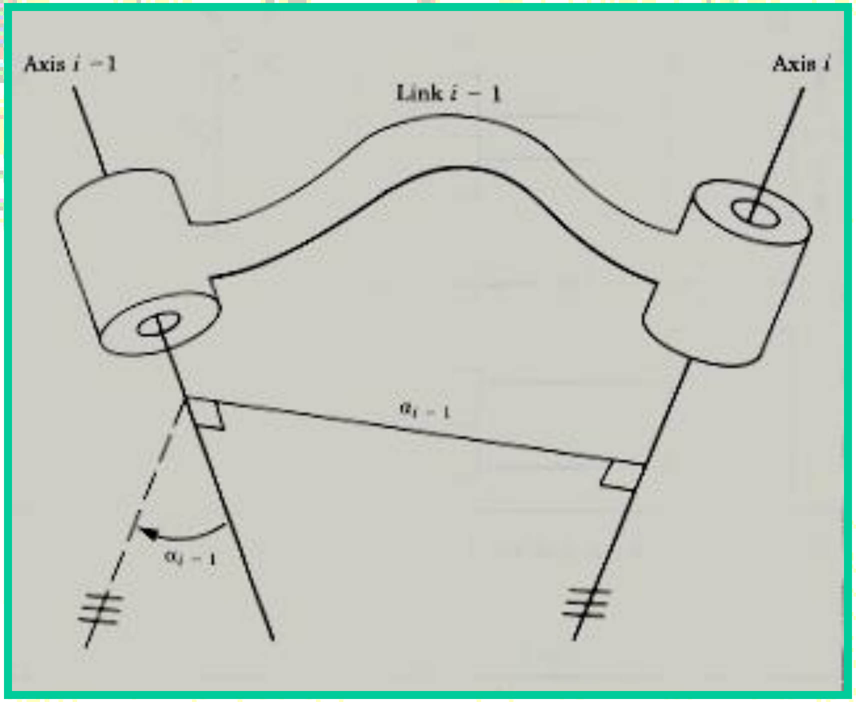
# Manipulator Kinematics

- **Kinematics Description of a Link:**

  - **A *Link* is considered only as a rigid body which defines the relationship between two neighboring joint axes of a manipulator.**

    - ✓ **Joint-axes are defined by lines in space as shown.**

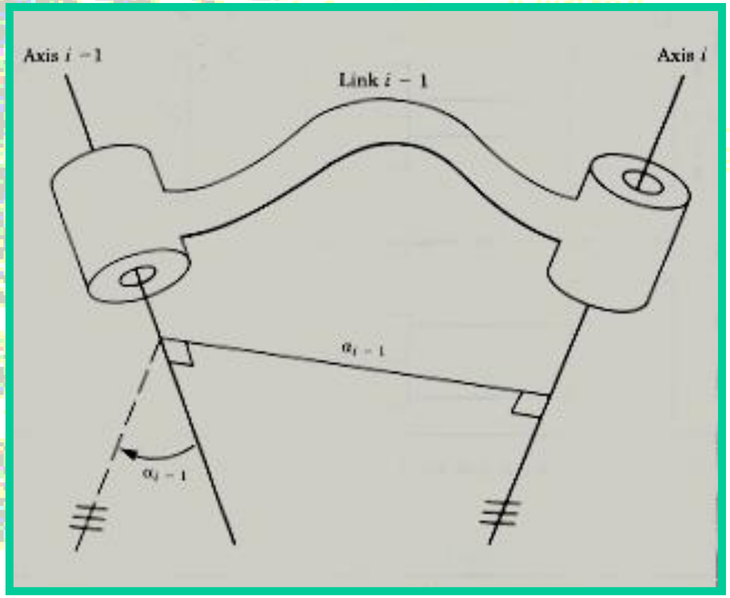
    - ✓ **How do we define the relationship between two lines in 3D-Space?**



# Manipulator Kinematics

## • Kinematics Description of a Link:

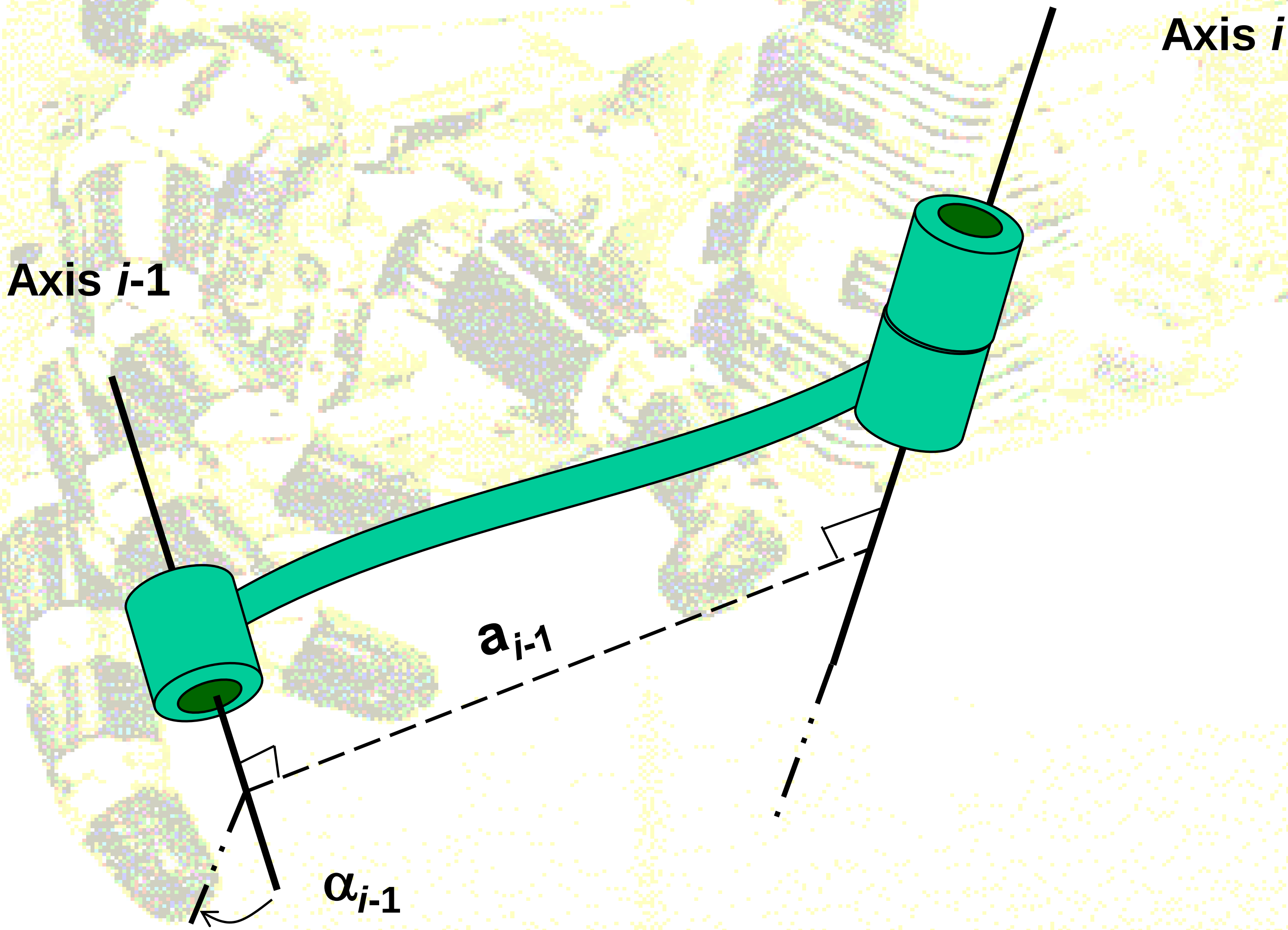
➢ **We need two quantities to define relative location of the two axes in a 3D-Space.**




1 **Distance between the lines along their common normal (Unique except when lines are parallel). This distance is called the "Link-Length"(طول رابط), $a_{i-1}$ measured from axis i-1 to axis i.**

2 **Angle between lines measured in the plane whose normal is the common normal. This angle is called the "Link-Twist"(پیچش رابط), $\alpha_{i-1}$ measured from axis i-1 to axis i by Right-Hand-Rule about the common normal. Note: If the axes intersect, then $a$ is zero, and $\alpha$ is still measured from axis i-1 to axis i.**

# Link Length and Twist



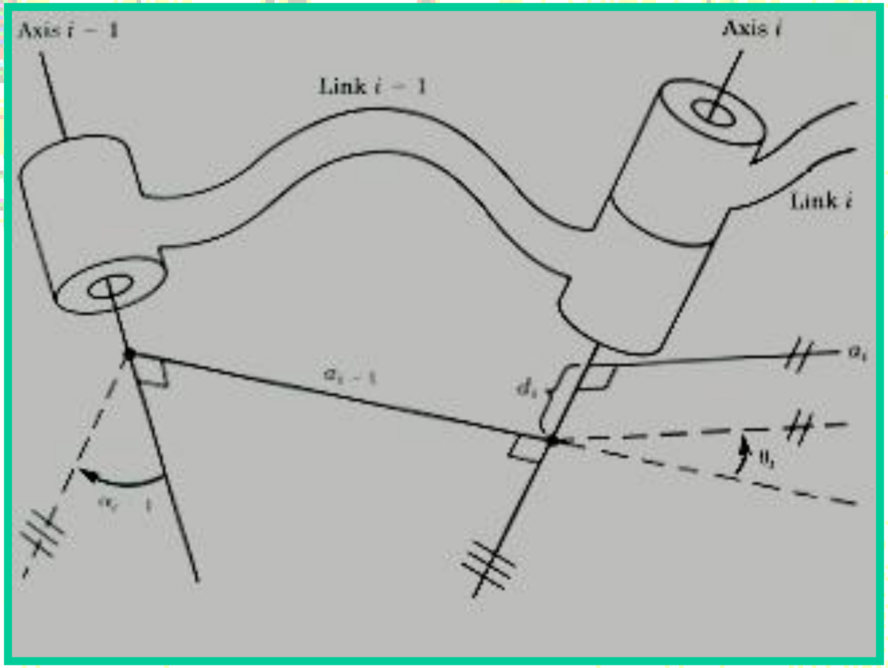
# Manipulator Kinematics

- **Link Connection Description (توصيف اتصال رابط):**

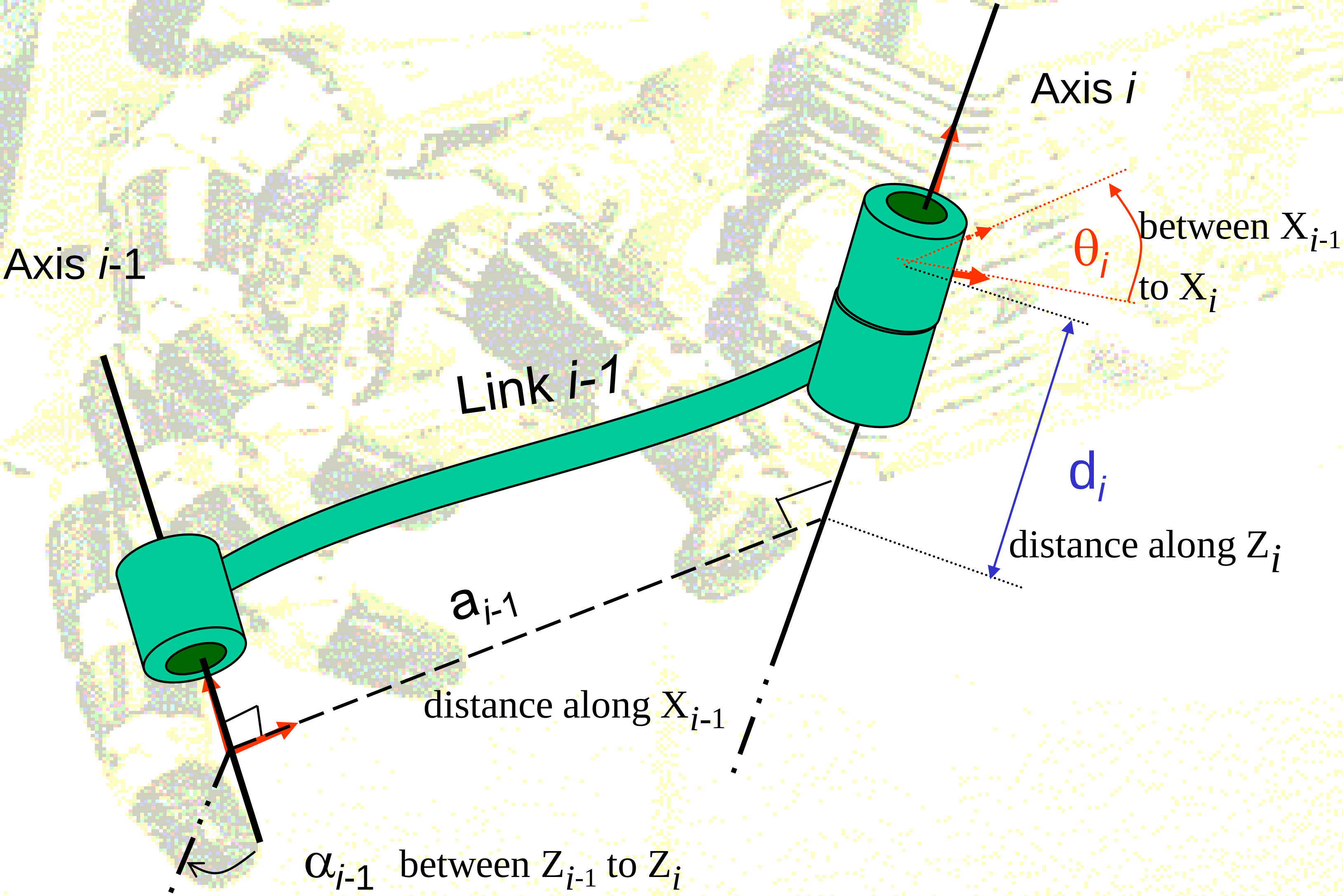
> **Links are connected in various ways, and issues such as joint strength, lubrication, bearings and gearing are usually considered at the design stage. However, for kinematical studies we only need two more quantities to completely define the relative position of two neighboring links.**

1. **The Distance between the two common normals "$d_i$" at joint-i. This distance is called the "Link-Offset" (انحراف رابط).**

2. **Angle of rotation about their common axis-i, between one link and its neighbor, "$\theta_i$". This angle is called the "Joint-Angle" (زاويه مفصلي). ("$\theta_i$" is the angle between $a_{i-1}$ and $a_i$ about axis-i).**



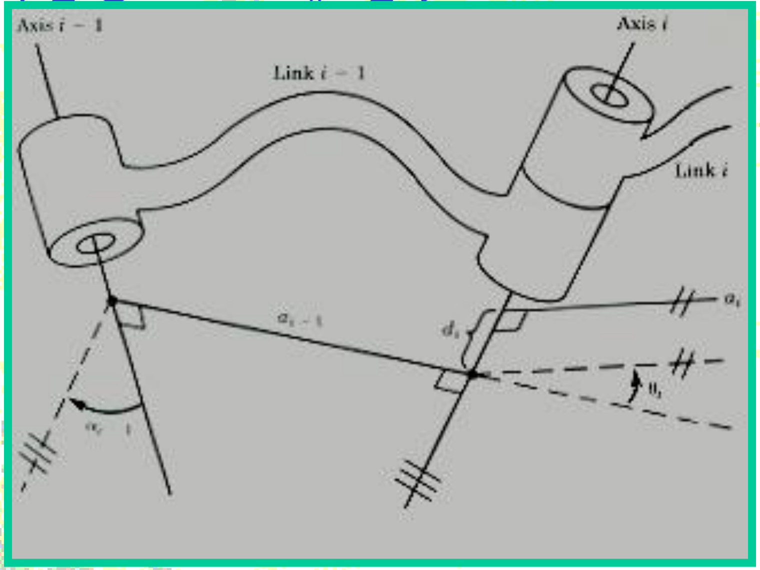
# Denavit-Hartenberg Parameters

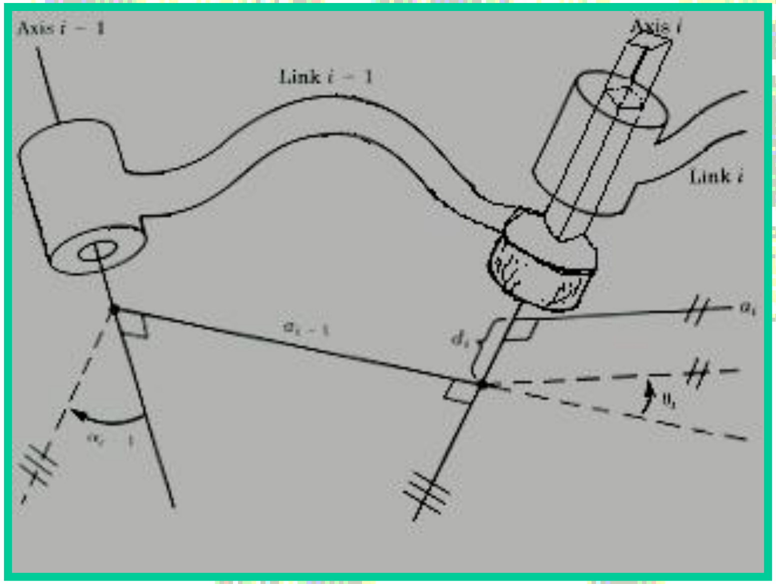


# Manipulator Kinematics

- **Link Connection Description (توصيف اتصال رابط):**

**For Revolute Joints: a, α, and d are all fixed, then "$\theta_i$" is the Joint Variable (متغیر مفصلی).**




**For Prismatic Joints: a, α, and θ are all fixed, then "$d_i$" is the Joint Variable (متغیر مفصلی).**

**These four parameters: (Link-Length≡ طول رابط ≡ $a_{i-1}$), (Link-Twist ≡ پیچش رابط ≡ $\alpha_{i-1}$), (Link-Offset ≡ انحراف رابط ≡ $d_i$), (Joint-Angle ≡ زاویه مفصلي ≡ $\theta_i$) are known as the *Denavit-Hartenberg Link Parameters*.**

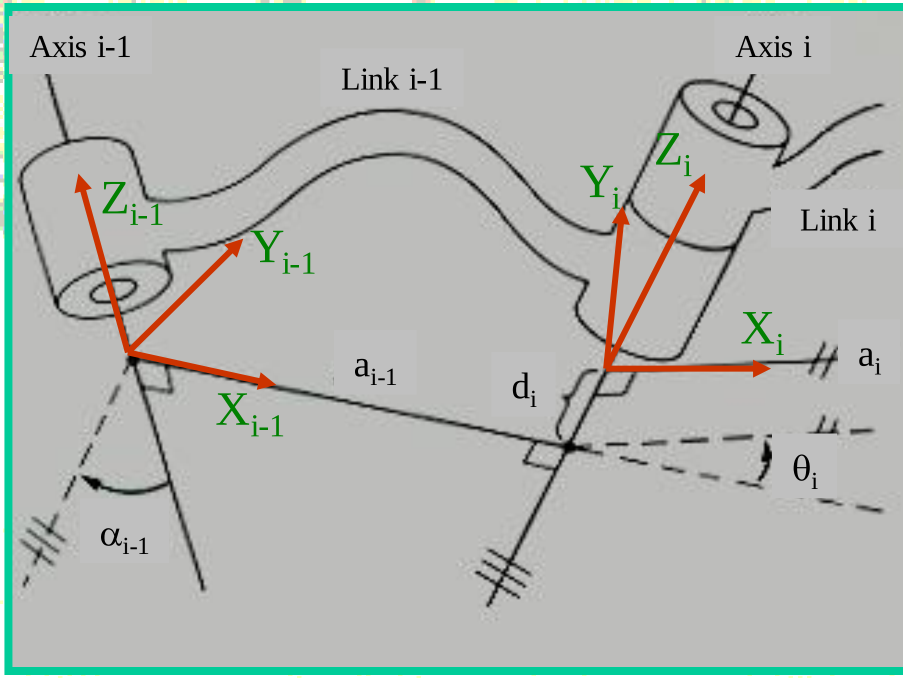
# Manipulator Kinematics

- **Affixing Frames to Links (قرارداد اتصال دستگاه به رابطها):**

➢ **To describe relative location of each link to its neighboring link, we shall attach a set of frames to each link in a manipulator in accordance to the following convention (frame {i} is rigidly attached to the link-i):**

❖ **Intermediate Links (رابطهای میانی):**

1. **The $Z_i$-axis of frame-{i}, called "$Z_i$", is coincident with the joint axis-i.**
2. **The origin of frame-{i} is located where the $a_i$-perpendicular intersects the "i-th" axis.**
3. **$X_i$-axis points along "$a_i$" in the direction from joint "i" to joint "i+1".**
4. **$Y_i$-axis is formed by the RHR to complete the "i-th" frame.**
5. **If the joint axes intersect, $a_i=0$, then $X_i$-axis is chosen normal to the plane of $Z_i$ and $Z_{i+1}$.** $(\hat{X}_i = \pm(\hat{Z}_i \times \hat{Z}_{i+1})$

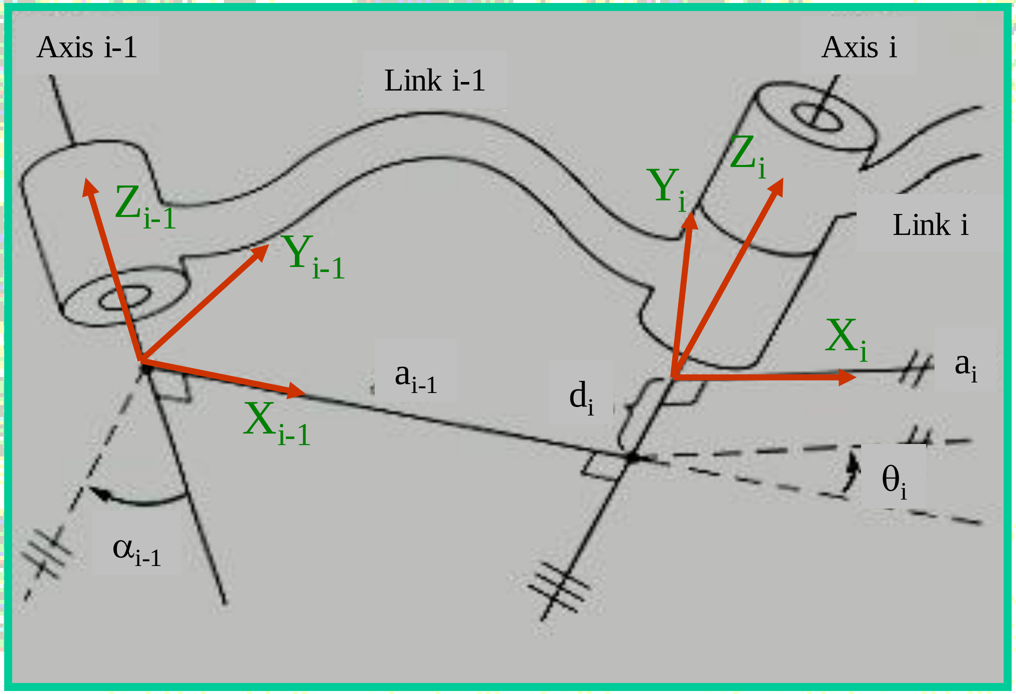


# Manipulator Kinematics

- Affixing Frames to Links (قرارداد اتصال دستگاه به رابطها):

➢ Intermediate Links (رابطهای میانی):

6. $a_{i-1}$ is the distance from $Z_{i-1}$ to $Z_i$ along $X_{i-1}$.
7. $\theta_i$ is the angle of rotation from $X_{i-1}$ to $X_i$ about $Z_i$.
8. $\alpha_{i-1}$ is the angle between $Z_{i-1}$ and $Z_i$ about $X_{i-1}$.
9. $d_i$ is the distance from $X_{i-1}$ to $X_i$ along $Z_i$.



# Manipulator Kinematics

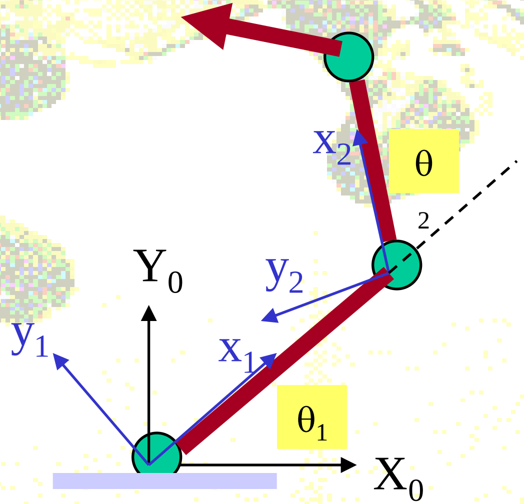
- Affixing Frames to Links (قرارداد اتصال دستگاه به رابطها):

➢ First and Last Links (رابطهای اول و آخر):

1. First:

   Attach a frame to the base of the robot (fixed) or link-0, and call it frame {0} (reference frame).

   *Note*: Frame {0} is arbitrary, therefore for simplicity choose frame {0} to be coincident with frame {1} when joint variable-1 (i.e. $\theta_1=0$) is zero. Using this convention, $a_0=0$, $\alpha_0=0$, and $d_1=0$ (if joint is *revolute*), and $\theta_1=0$ (if joint is *prismatic*).



# Manipulator Kinematics

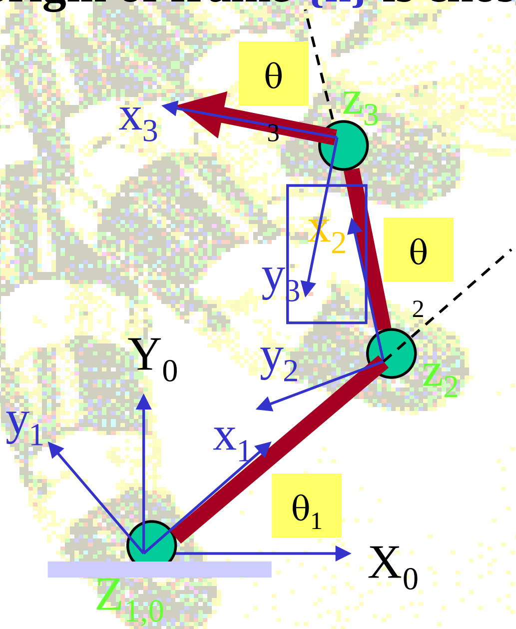
- **Affixing Frames to Links (قرارداد اتصال دستگاه به رابطها):**

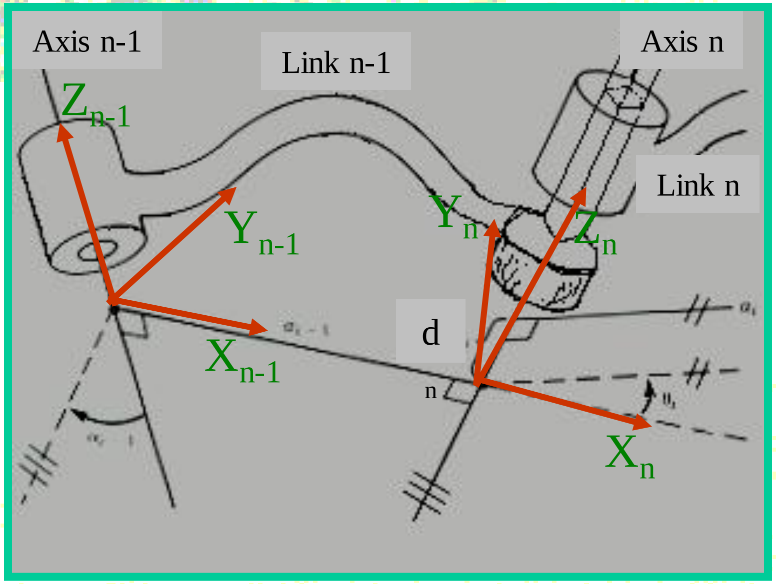
➢ **First and Last Links (رابطهای اول و آخر):**

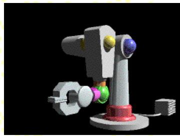
2. **Last:**

   **Revolute:** If joint-n is revolute, choose the direction of $X_n$ such that it aligns with $X_{n-1}$ when $\theta_n=0$, and the origin of frame {n} is chosen so that $d_n=0$.

   **Prismatic:** If joint-n is prismatic, choose the direction of $X_n$ such that $\theta_n=0$ and the origin of frame {n} is chosen at the intersection of $X_{n-1}$ and $Z_n$ when $d_n=0$.



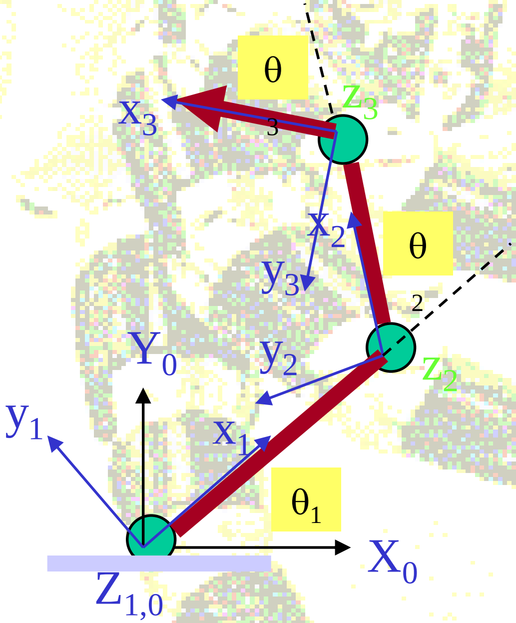



# Manipulator Kinematics

- **Affixing Frames to Links (قرارداد اتصال دستگاه به رابطها):**

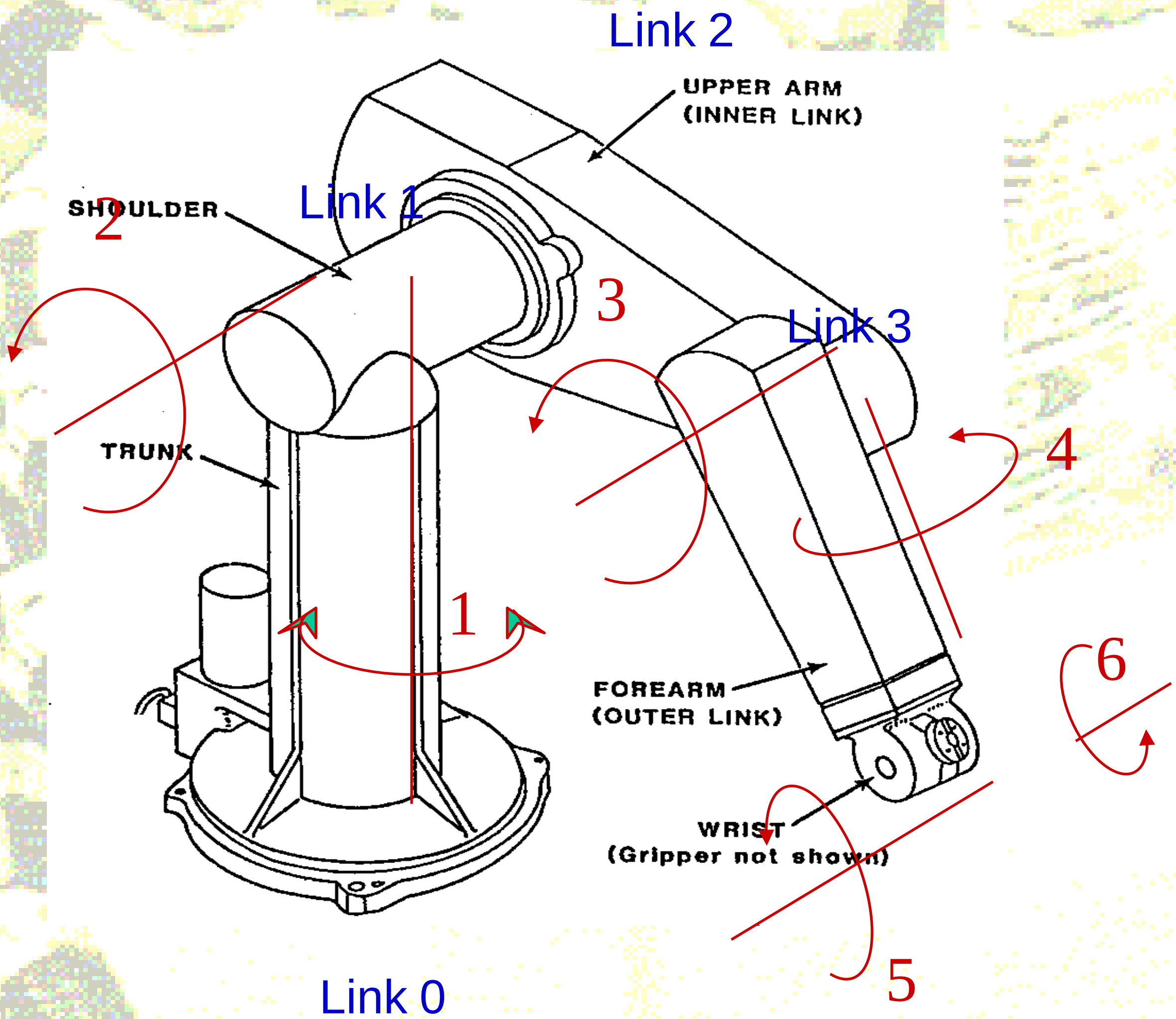
➢ **Example: A 3-link planar manipulator**

1. **Define the reference frame {0} so that it aligns with frame {1} when $\theta_1=0$ .**
2. **Since the arm is planar, all Z-axes are parallel. (No link offsets ⇔ all $d_i$ =0).**
3. **Since all joints are revolute, when their values are zero, all X-axes must align.**

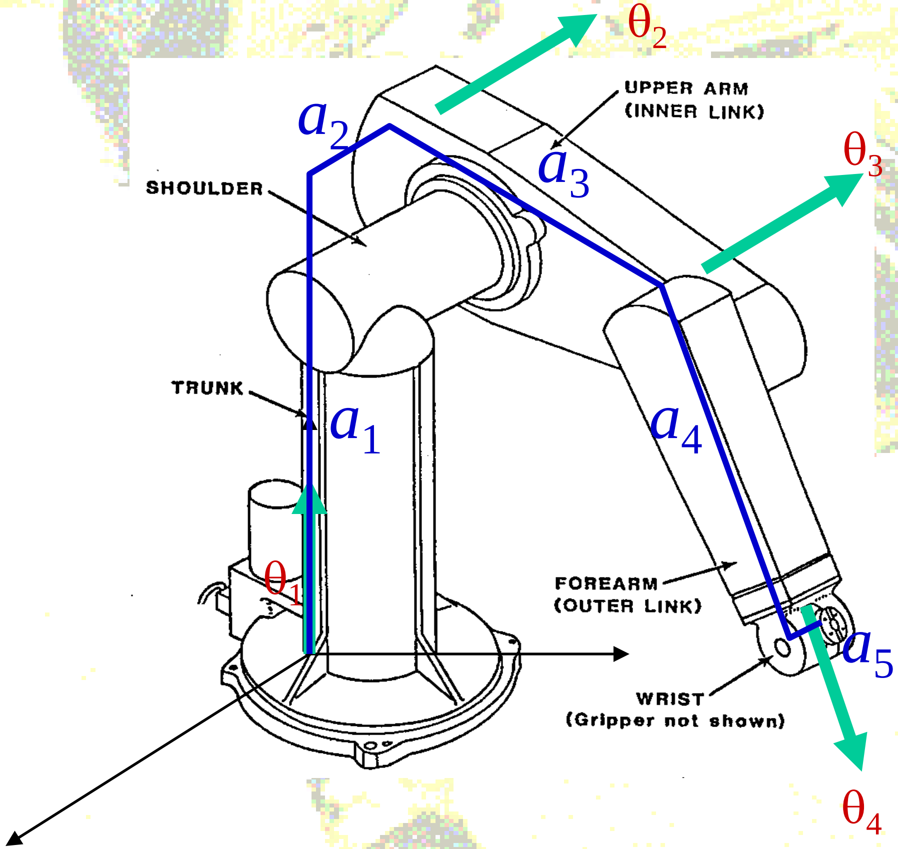


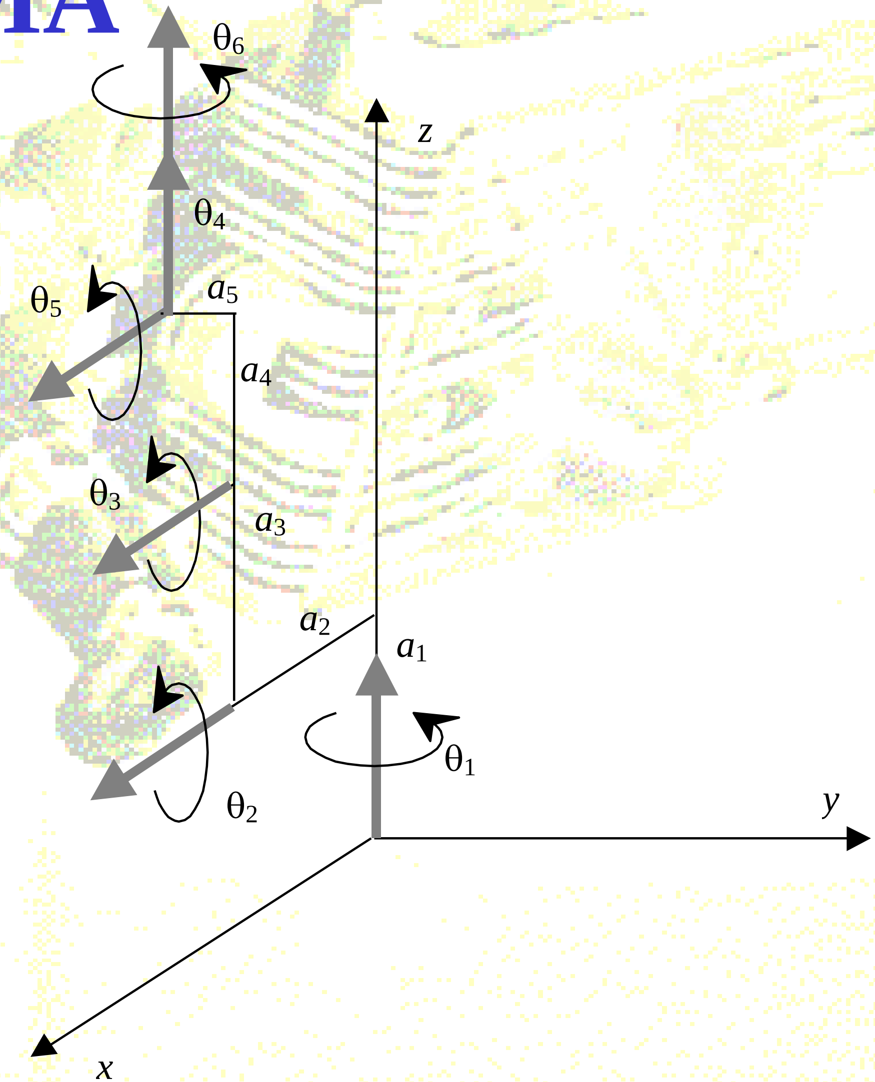

| Joint-i | $\theta_i$ | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | $\alpha_0=0$ | $a_0=0$ | $d_1=0$ |
| 2 | $\theta_2$ | $\alpha_1=0$ | $a_1=L_1$ | $d_2=0$ |
| 3 | $\theta_3$ | $\alpha_2=0$ | $a_2=L_2$ | $d_3=0$ |

# Kinematic Modeling

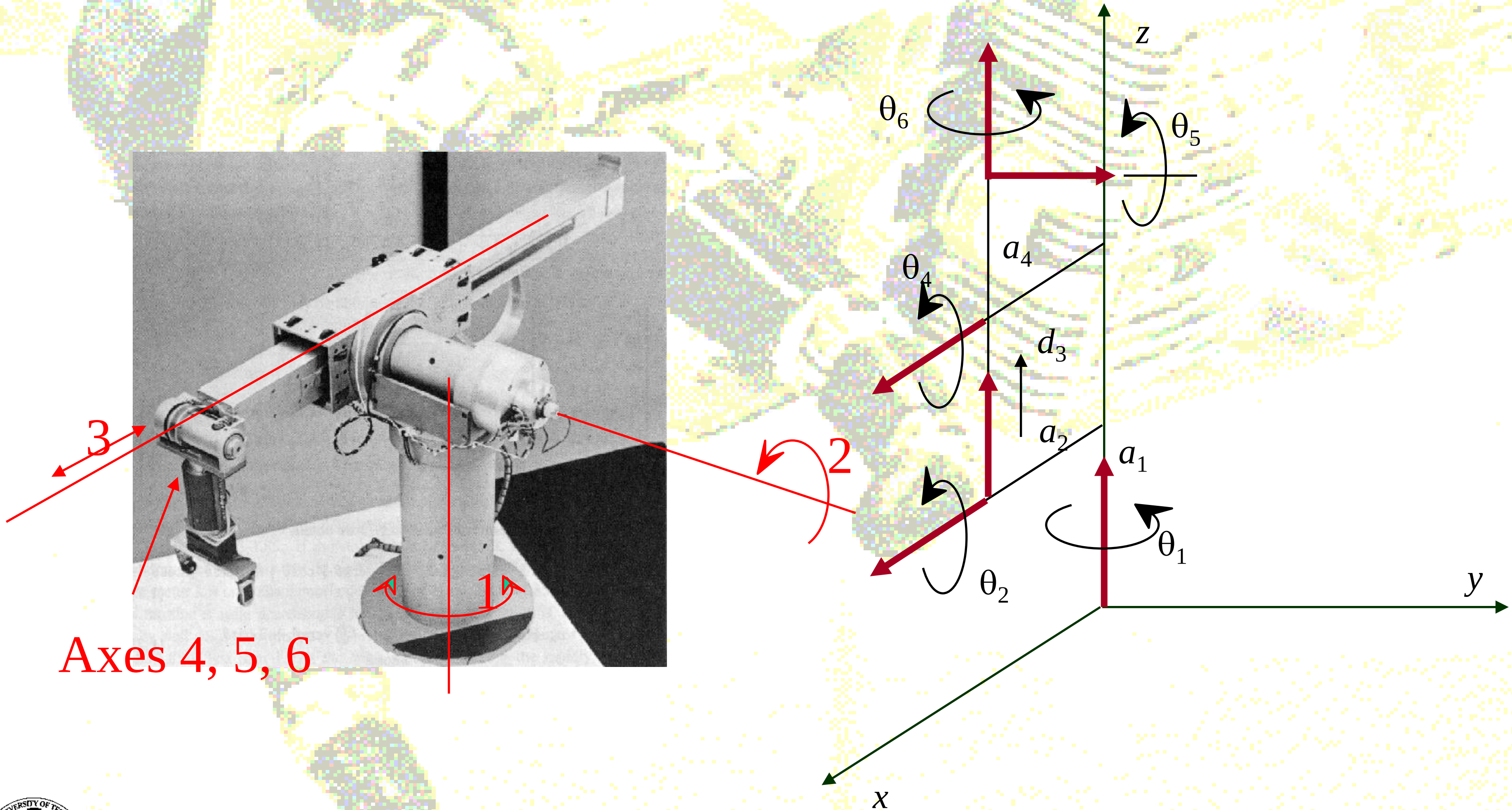


# Offsets and Home Position for the PUMA

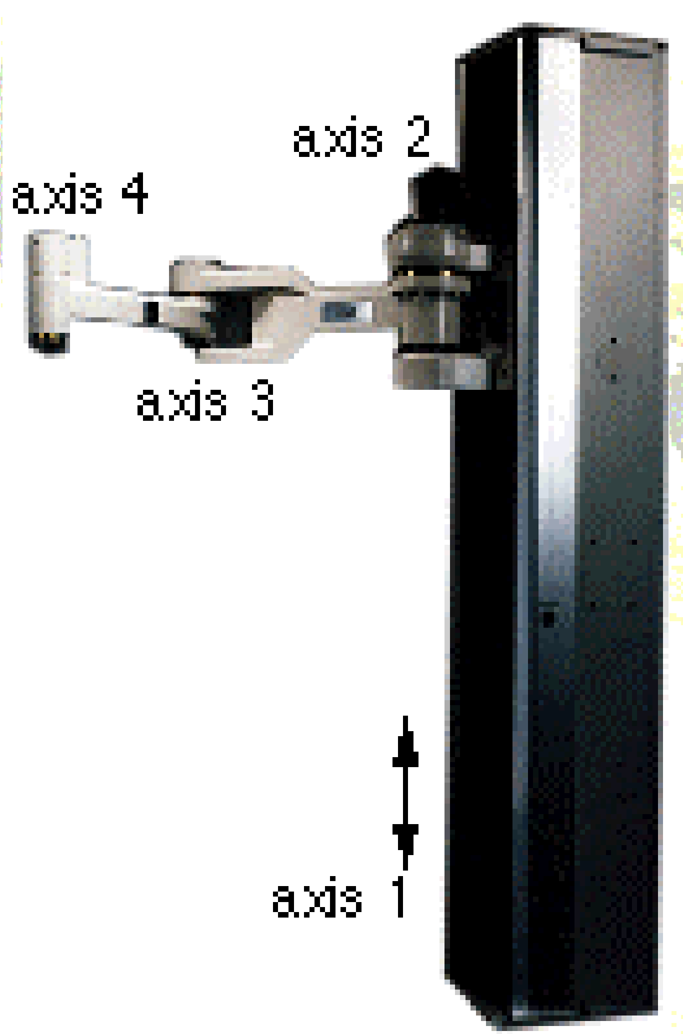


# Offsets and Zero Position for the Stanford Arm

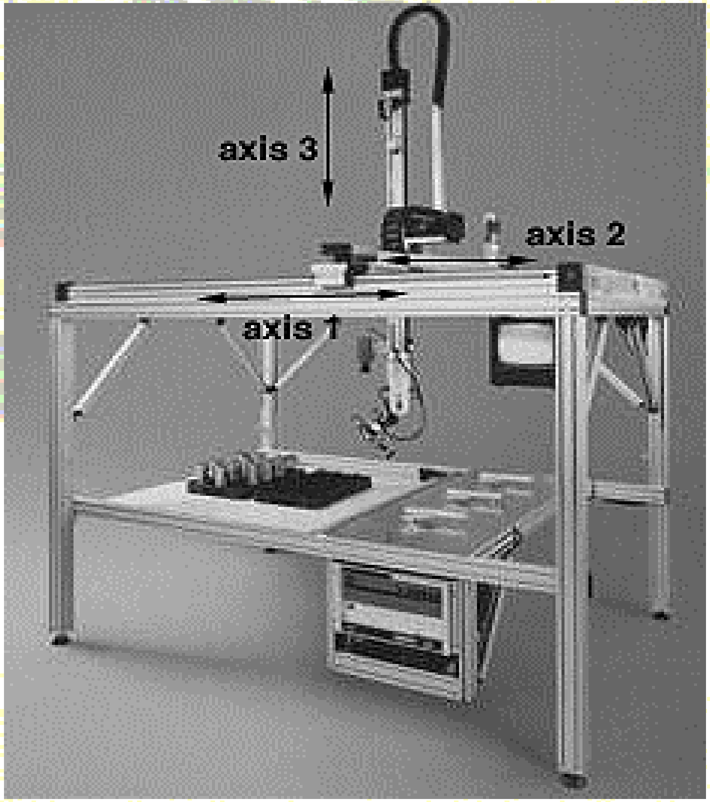


# Degrees-of-Freedom




Adept Robot:  PRRR manipulator

The G365 Gantry robot manipulator (CRS Robotics)

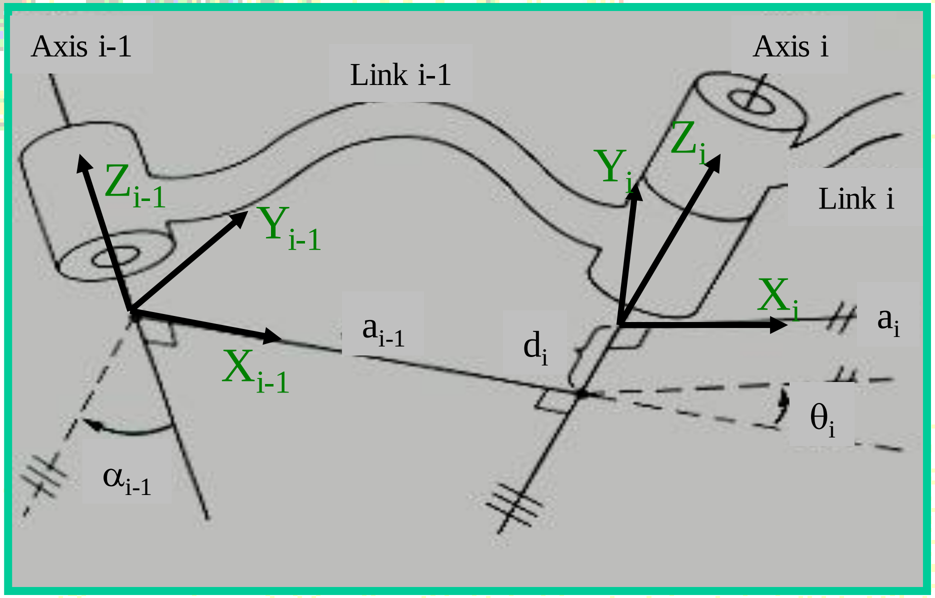


# Manipulator Kinematics

- **The "T" Transformation (ماتریس تبدیل–تی):**

  - **We shall now derive the *General form of Transformations* which relates frames attached to neighboring links.**
  - **In general, two neighboring links may be shown as follows:**
  - **We wish to determine the transformation which defines frame {i} relative to the frame {i-1}.**

$$ {}^{i-1}_{i}T = ? \equiv f(a_{i-1}, \alpha_{i-1}, d_i, \theta_i) $$

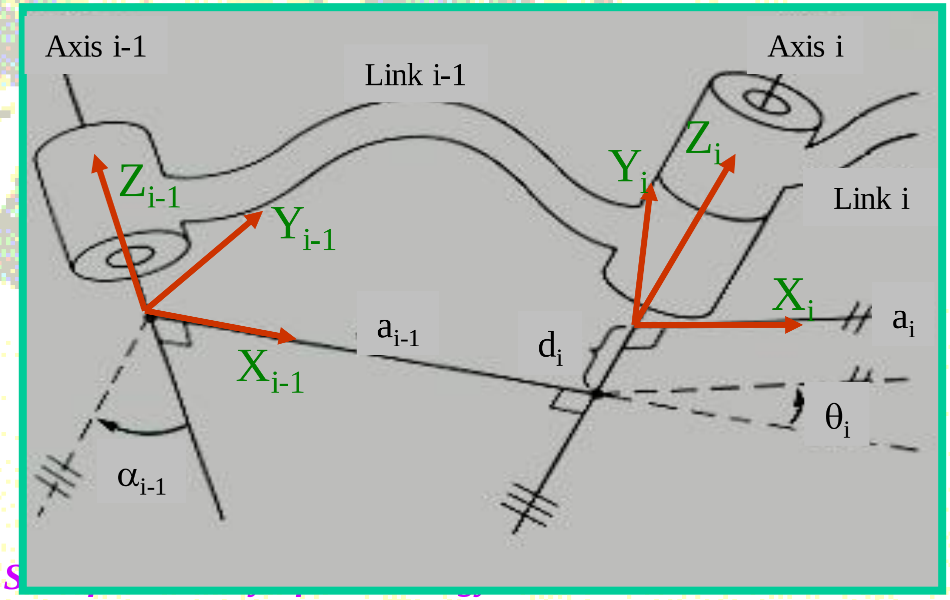



  - **One can easily align frame {i-1}on frame {i} by 4-simple transformations as follows:**

# Manipulator Kinematics

- **The "T" Transformation (ماتریس تبدیل–تی):**

1. **Rotate frame {i-1} about $X_{i-1}$ axis by $\alpha_{i-1}$ to make the $Z_{i-1}$ in the same direction as $Z_i$. $Rot(X_{i-1}, \alpha_{i-1})$**
2. **Translate along $X_{i-1}$ axis by $a_{i-1}$ to bring the two origins on the same axis $Z_i$. $Trans(X_{i-1}, a_{i-1})$**
3. **Rotate about $Z_i$ axis by $\theta_i$ to make $X_{i-1}$ in the same direction as $X_i$. $Rot(Z_i, \theta_i)$**
4. **Translate along $Z_i$ axis by $d_i$ to make the two frames completely coincide. $Trans(Z_i, d_i)$**



# Manipulator Kinematics

- **The "T" Transformation (ماتریس تبدیل–تی):**

**Combining all transformations results in:**

$$ {}^{i-1}_{\ \ i}T = Rot(\hat{X}_{i-1},\alpha_{i-1})Trans(\hat{X}_{i-1},a_{i-1})Rot(\hat{Z}_i,\theta_i)Trans(\hat{Z}_i,d_i) \equiv $$

$$ \equiv \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -S\alpha_{i-1} d_i \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & C\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

# Manipulator Kinematics

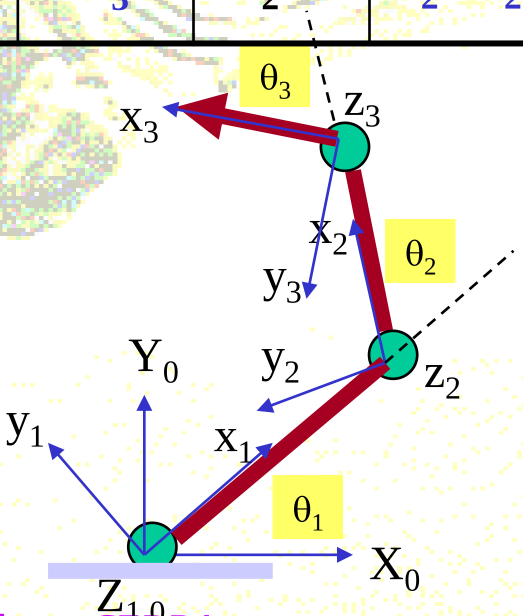
- **Affixing Frames to Links (قرارداد اتصال دستگاه به رابطها):**
  - **Example: The 3-link planar manipulator**

$$ {}^{0}_{1}T = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}_{2}T = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & \ell_1 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}T = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & \ell_2 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

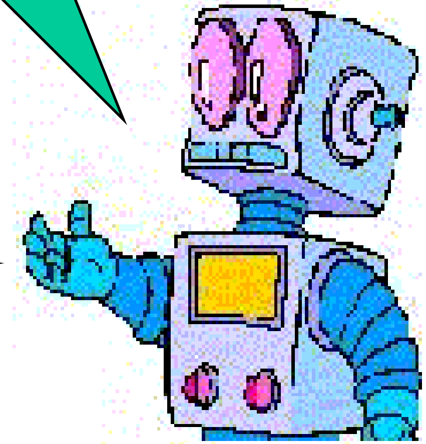
| Joint-i | $\theta_i=0$ | $\alpha_{i-1}$ | $a_{i-1}$ | $d_i$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | $\alpha_0=0$ | $a_0=0$ | $d_1=0$ |
| 2 | $\theta_2$ | $\alpha_1=0$ | $a_1=L_1$ | $d_2=0$ |
| 3 | $\theta_3$ | $\alpha_2=0$ | $a_2=L_2$ | $d_3=0$ |



# Direct/Forward Kinematics

Where is my hand?

Direct Kinematics:
HERE!



# Manipulator Kinematics

- **Forward Kinematics (سینماتیک مستقیم):**

**Given the joint variables ($\theta_1$, $\theta_2$, …), compute the Position and Orientation of the last link of the manipulator arm relative to the base frame?**

**Given:**

$$ {}^{i-1}_{i}T, \qquad i = 1, \ldots, n $$

$$ {}^{0}_{n}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T \ldots\ldots {}^{n-1}_{n}T $$

**Where:** ${}^{0}_{n}T$ **is function of n joint variables, and represents the Cartesian position & orientation of the last link relative to base frame.**