



**School of Mechanical Engineering
Sharif University of Technology**

Convection Heat Transfer

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Lecture #2

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Convection Heat Transfer



Fundamentals and Basic Concepts

Convection Heat Transfer



Summary of first session:

✓ Basic concepts and applications

✓ Fundamental governing equations: Mass conservation and Momentum principle

This session: First and second laws of Thermodynamics

Convection Heat Transfer



FIRST LAW OF THERMODYNAMICS

The importance and usage
of First law of
Thermodynamics

In case the natural flow is driven by the heat administered to the flowing fluid.

In all cases, however, the heat transfer part of the convection problem requires a solution for the temperature distribution through the flow, especially in the close vicinity of the solid walls bathed by the heat-carrying fluid stream.

$$\begin{aligned} & \left(\begin{array}{l} \text{rate of energy} \\ \text{accumulation in the} \\ \text{control volume} \end{array} \right)_1 \\ &= \left(\begin{array}{l} \text{net transfer of} \\ \text{energy by fluid flow} \end{array} \right)_2 + \left(\begin{array}{l} \text{net heat transfer} \\ \text{by conduction} \end{array} \right)_3 \\ &+ \left(\begin{array}{l} \text{rate of internal} \\ \text{heat generation (e.g.,} \\ \text{electrical power} \\ \text{dissipation)} \end{array} \right)_4 - \left(\begin{array}{l} \text{net work transfer} \\ \text{from the control} \\ \text{volume to its} \\ \text{environment} \end{array} \right)_5 \end{aligned}$$

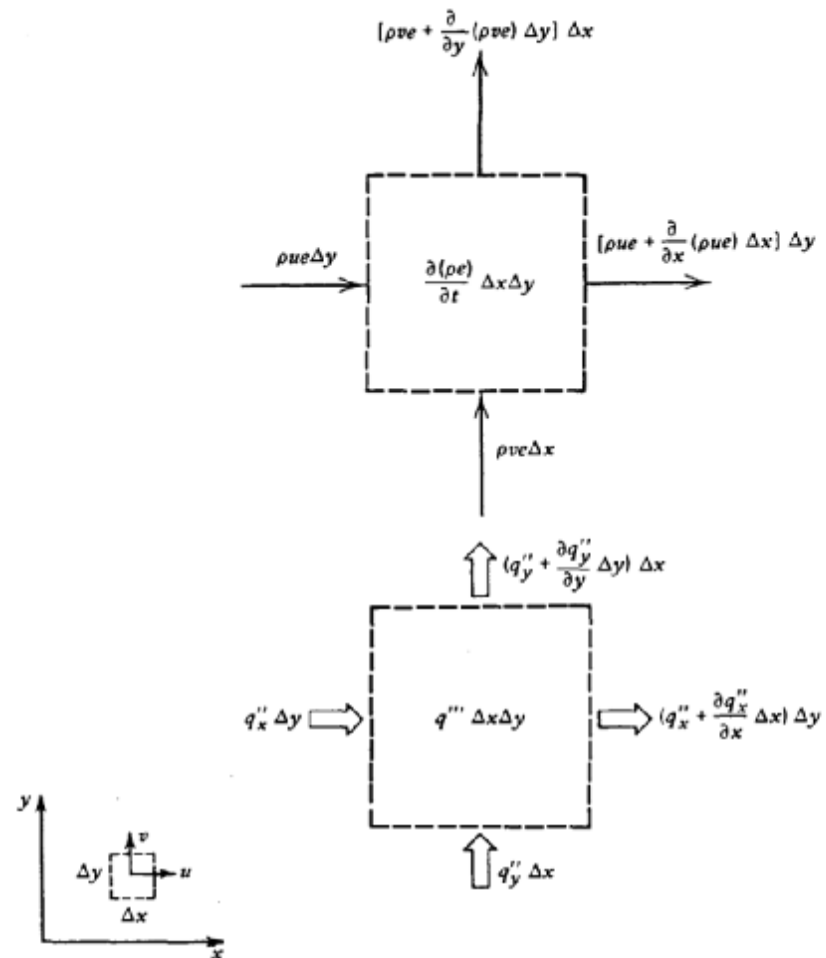
(25-a)



Convection Heat Transfer

FIRST LAW OF THERMODYNAMICS

First law of thermodynamics applied to a control volume in two-dimensional flow

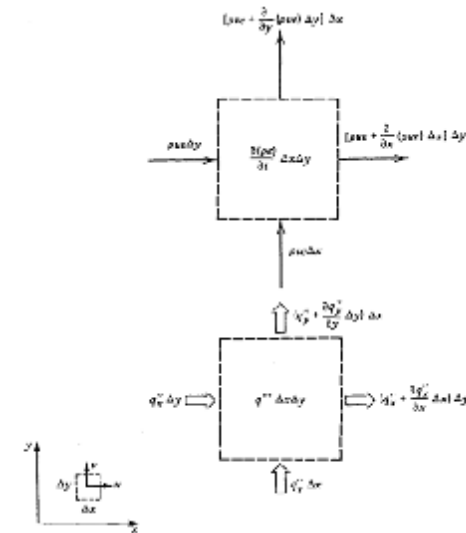




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FIRST LAW OF THERMODYNAMICS

According to the energy flow diagrams sketched in the Figure, the groups of the above terms are



$$\left\{ \frac{dE_{cv}}{dt} \right\}_1 = [\sum \dot{m}_i h_i - \sum \dot{m}_e h_e]_2 + [\sum q'' \Delta x \Delta y]_3 + [q_i]_4 - [W_{cv}]_5$$

The origin of the dissipation rate term $\{ \bullet \}_5$ lies in the work transfer affected by the normal and tangential stresses

$$\begin{aligned} \{ \bullet \}_1 &= \Delta x \Delta y \frac{\partial}{\partial t} (\rho e) \\ \{ \bullet \}_2 &= -(\Delta x \Delta y) \left[\frac{\partial}{\partial x} (\rho u e) + \frac{\partial}{\partial y} (\rho v e) \right] \\ \{ \bullet \}_3 &= -(\Delta x \Delta y) \left(\frac{\partial q_x''}{\partial x} + \frac{\partial q_y''}{\partial y} \right) \\ \{ \bullet \}_4 &= (\Delta x \Delta y) q''' \\ \{ \bullet \}_5 &= (\Delta x \Delta y) \left(\sigma_x \frac{\partial u}{\partial x} - \tau_{xy} \frac{\partial u}{\partial y} + \sigma_y \frac{\partial v}{\partial y} - \tau_{yx} \frac{\partial v}{\partial x} \right) \\ &\quad + (\Delta x \Delta y) \left(u \frac{\partial \sigma_x}{\partial x} - u \frac{\partial \tau_{xy}}{\partial y} + v \frac{\partial \sigma_y}{\partial y} - v \frac{\partial \tau_{yx}}{\partial x} \right)^* \end{aligned}$$

(25-b)



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FIRST LAW OF THERMODYNAMICS

In the $\{\bullet\}_5$ expression, the eight terms have been separated into two groups.

$$\{\bullet\}_5 = (\Delta x \Delta y) \left(\sigma_x \frac{\partial u}{\partial x} - \tau_{xy} \frac{\partial u}{\partial y} + \sigma_y \frac{\partial v}{\partial y} - \tau_{yx} \frac{\partial v}{\partial x} \right) + (\Delta x \Delta y) \left(u \frac{\partial \sigma_x}{\partial x} - u \frac{\partial \tau_{xy}}{\partial y} + v \frac{\partial \sigma_y}{\partial y} - v \frac{\partial \tau_{yx}}{\partial x} \right) *$$

Represents the change in kinetic energy of the fluid packet; in the present treatment, this change **is considered negligible relative to the internal energy change $d(re)/dt$** appearing in term $\{\bullet\}_1$

$$\rho(D/Dt)(u^2 + v^2)/2$$

Assembling expressions (25) into the energy conservation statement and using constitutive relations, we obtain:

$$\rho \frac{De}{Dt} + e \left(\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} \right) = -\nabla \cdot \mathbf{q}'' + q''' - P \nabla \cdot \mathbf{v} + \mu \Phi$$

(26)



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FIRST LAW OF THERMODYNAMICS

$\equiv 0$; Due to mass conservation

$$\rho \frac{De}{Dt} + e \left(\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} \right) = -\nabla \cdot \mathbf{q}'' + q''' - P \nabla \cdot \mathbf{v} + \mu \Phi \quad (26)$$

Heat flux vector

Viscous dissipation function

In the special case where the flow can be modeled as incompressible and two-dimensional, the viscous dissipation function reduces to

$$\Phi = 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \quad (27)$$



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FIRST LAW OF THERMODYNAMICS

To express Eq. (26) in terms of enthalpy, we use the thermodynamics definition: $h = e + (1/\rho)P$

$$\frac{Dh}{Dt} = \frac{De}{Dt} + \frac{1}{\rho} \frac{DP}{Dt} - \frac{P}{\rho^2} \frac{D\rho}{Dt} \quad (28)$$

we can express the directional heat fluxes q''_x and q''_y in terms of the local temperature gradients (we invoke the Fourier law of heat conduction)

$$\mathbf{q}'' = -k \nabla T \quad (29)$$

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FIRST LAW OF THERMODYNAMICS

Combining Eqs. 26, 28 and 29:

$\equiv 0$; Due to mass conservation

$$\rho \frac{Dh}{Dt} = \nabla \cdot (k \nabla T) + q''' + \frac{DP}{Dt} + \mu\Phi - \frac{P}{\rho} \left(\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{v} \right) \quad (30)$$

FIRST LAW OF THERMODYNAMICS

$$\rho \frac{Dh}{Dt} = \nabla \cdot (k \nabla T) + q''' + \frac{DP}{Dt} + \mu\Phi \quad (31)$$

Now we intend to express the energy equation in terms of temperature

$$dh = T ds + \frac{1}{\rho} dP \quad (32)$$

where T is the absolute temperature and ds the specific entropy change.

$$ds = \left(\frac{\partial s}{\partial T} \right)_P dT + \left(\frac{\partial s}{\partial P} \right)_T dP \quad (33)$$

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FIRST LAW OF THERMODYNAMICS

From the Maxwell's relations [3, p. 173], we have:

$$\left(\frac{\partial s}{\partial P}\right)_T = -\left[\frac{\partial(1/\rho)}{\partial T}\right]_P = \frac{1}{\rho^2}\left(\frac{\partial\rho}{\partial T}\right)_P = -\frac{\beta}{\rho} \quad (34)$$

Where β is the coefficient of thermal expansion.

$$\beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_P \quad (35)$$

Next slide shows the useful Thermodynamic relations



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FIRST LAW OF THERMODYNAMICS

Summary of Thermodynamic relations

	Internal Energy $du = T ds - P dv$	Enthalpy $dh = T ds + v dP$	Entropy $ds = \frac{1}{T} du + \frac{P}{T} dv$
Pure substance	$du = c_v dT + \left[T \left(\frac{\partial P}{\partial T} \right)_v - P \right] dv$	$dh = c_p dT + \left[-T \left(\frac{\partial v}{\partial T} \right)_p + v \right] dP$	$ds = \frac{c_p}{T} dT - \left(\frac{\partial v}{\partial T} \right)_p dP$ $= \frac{c_v}{T} dT + \left(\frac{\partial P}{\partial T} \right)_v dv$
Ideal gas	$du = c_v dT$	$dh = c_p dT$	$ds = c_p \frac{dT}{T} - R \frac{dP}{P}$ $= c_v \frac{dT}{T} + R \frac{dv}{v}$ $= c_v \frac{dP}{P} + c_p \frac{dv}{v}$
Incompressible liquid	$du = c dT$	$dh = c dT + v dP$	$ds = c \frac{dT}{T}$

Source: Ref. 3.

^aAccording to the classical thermodynamics notation, v is the specific volume, $v = 1/\rho$, and u is the internal energy (e in the text).

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FIRST LAW OF THERMODYNAMICS

From Table we have:

$$\left(\frac{\partial s}{\partial T}\right)_P = \frac{c_P}{T} \quad (36)$$

Eqs. (32)-(36) state that

$$dh = c_P dT + \frac{1}{\rho} (1 - \beta T) dP \quad (37)$$

left-hand side of the Energy Equation (31) is

$$\rho \frac{Dh}{Dt} = \rho c_P \frac{DT}{Dt} + (1 - \beta T) \frac{DP}{Dt} \quad (38)$$

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FIRST LAW OF THERMODYNAMICS

The "temperature" formulation of the first law of thermodynamics is:

$$\rho c_p \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + q''' + \beta T \frac{DP}{Dt} + \mu \Phi \quad (39)$$

For special case: ideal gas($b=1/T$), we have:

$$\rho c_p \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + q''' + \frac{DP}{Dt} + \mu \Phi \quad (40)$$

Incompressible liquid ($b= 0$):

$$\rho c \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + q''' + \mu \Phi \quad (41)$$

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FIRST LAW OF THERMODYNAMICS

Most of the convection problems obey an even simpler model: namely, constant fluid conductivity k , zero internal heat generation q''' , negligible viscous dissipation $\mu\Phi$, and negligible compressibility effect βT . The energy equation for this model is:

$$\rho c_p \frac{DT}{Dt} = k \nabla^2 T \quad (42)$$

Simplified energy equation for Cartesian coordinates (X, Y, Z) :

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (43-a)$$

Simplified energy equation for Cylindrical coordinates (r, Θ, z) :

$$\begin{aligned} \rho c_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) \\ = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right] \end{aligned} \quad (43-b)$$

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FIRST LAW OF THERMODYNAMICS

Simplified energy equation for Spherical coordinates (r, Φ, Θ) :

$$\rho c_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\phi}{r} \frac{\partial T}{\partial \phi} + \frac{v_\theta}{r \sin \phi} \frac{\partial T}{\partial \theta} \right) = k \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \phi} \frac{\partial}{\partial \phi} \left(\sin \phi \frac{\partial T}{\partial \phi} \right) + \frac{1}{r^2 \sin^2 \phi} \frac{\partial^2 T}{\partial \theta^2} \right] \quad (43-c)$$

If the fluid can be modeled thermodynamically as an incompressible liquid, then the specific heat at constant pressure C_p is replaced by The specific heat of the incompressible liquid, c

Convection Heat Transfer



Assignment #2

Derive the energy equation in the spherical coordinate system

Due date: One week after this session

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FIRST LAW OF THERMODYNAMICS

When dealing with extremely viscous flows of the type encountered in lubrication problems or the piping of crude oil, the above model is improved by taking into account the internal heating due to viscous dissipation.

$$\rho c_p \frac{DT}{Dt} = k \nabla^2 T + \mu \Phi \quad (44)$$

In three dimensions, the general viscous dissipation function can be expressed as follows:

$$\begin{aligned} \Phi = & 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] \\ & + \left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right)^2 \right] \\ & - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2 \end{aligned} \quad (45-a)$$

**Cartesian
Coordinates**

Convection Heat Transfer



FIRST LAW OF THERMODYNAMICS

**Cylindrical
Coordinates**

$$\begin{aligned} \Phi = 2 & \left[\left(\frac{\partial v_r}{\partial r} \right)^2 + \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right)^2 + \left(\frac{\partial v_z}{\partial z} \right)^2 \right. \\ & + \frac{1}{2} \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right)^2 + \frac{1}{2} \left(\frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \right)^2 \\ & \left. + \frac{1}{2} \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right)^2 - \frac{1}{3} (\nabla \cdot \mathbf{v})^2 \right] \end{aligned}$$

(45-b)

**Spherical
Coordinates**

$$\begin{aligned} \Phi = 2 & \left\{ \left[\left(\frac{\partial v_r}{\partial r} \right)^2 + \left(\frac{1}{r} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} \right)^2 + \left(\frac{1}{r \sin \phi} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} + \frac{v_\phi \cot \phi}{r} \right)^2 \right] \right. \\ & + \frac{1}{2} \left[r \frac{\partial}{\partial r} \left(\frac{v_\phi}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \phi} \right]^2 + \frac{1}{2} \left[\frac{\sin \phi}{r} \frac{\partial}{\partial \phi} \left(\frac{v_\theta}{r \sin \phi} \right) + \frac{1}{r \sin \phi} \frac{\partial v_\theta}{\partial \theta} \right]^2 \\ & \left. + \frac{1}{2} \left[\frac{1}{r \sin \phi} \frac{\partial v_r}{\partial \theta} + r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \right]^2 \right\} - \frac{2}{3} (\nabla \cdot \mathbf{v})^2 \end{aligned}$$

(45-c)

If the density does not vary significantly through the flow field, $\nabla \cdot \mathbf{v} = 0$ and the last term in each of the expressions (45) vanishes.

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SECOND LAW OF THERMODYNAMICS

❖ The second law is the basis for much of the engineering Motive for formulating and solving convection problems.

❖ For example, in the development of know how for the heat exchanger industry, we strive for **improved thermal contact** (enhanced heat transfer) and **reduced pump power loss** in order to improve the thermodynamic efficiency of the heat exchanger. **Good heat exchanger design** means, ultimately, efficient Thermodynamic performance, that is, **minimum generation of entropy or minimum destruction of exergy in the power/refrigeration system incorporating the heat.**

Convection Heat Transfer



SECOND LAW OF THERMODYNAMICS

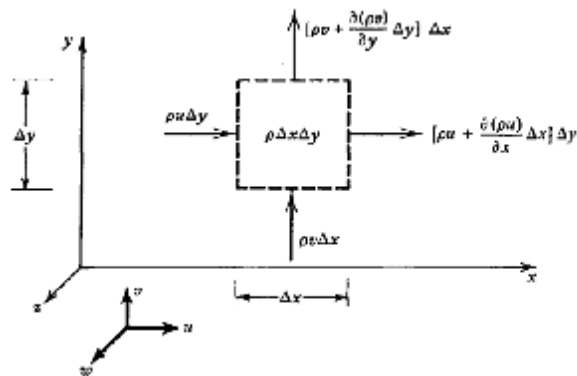
The second law of thermodynamics states that all real-life processes are irreversible: In the case of a control volume, as in Fig. 1, this statement is:

$$\frac{\partial S_{cv}}{\partial t} \geq \sum \frac{q_i}{T_i} + \sum_{\text{inlet ports}} \dot{m}s - \sum_{\text{outlet ports}} \dot{m}s \quad (46)$$

Instantaneous entropy inventory of the control volume

The absolute temperature of the boundary crossed by the heat transfer interaction q_i

The entropy flows (streams) into and out of the control volume



Convection Heat Transfer



SECOND LAW OF THERMODYNAMICS

The irreversibility of the process is measured by the strength of the inequality sign in eq. (46), or by the entropy generation rate S_{gen} defined as :

$$S_{gen} = \frac{\partial S_{cv}}{\partial t} - \sum \frac{q_i}{T_i} - \sum_{\text{inlet ports}} \dot{m}s + \sum_{\text{outlet ports}} \dot{m}s \geq 0 \quad (47)$$

It is easy to show that the rate of destruction of useful work in an engineering system, W_{lost} , is directly proportional to the rate of entropy generation:

$$W_{lost} = T_0 S_{gen} \quad (48)$$

absolute temperature of the ambient
temperature reservoir

Convection Heat Transfer



SECOND LAW OF THERMODYNAMICS

the second law (47) may be applied to a finite-size control volume $\Delta x \Delta y \Delta z$ at an arbitrary point (x, y, z) in a flow field. Thus, the rate of entropy generation per unit time and per unit volume S_{gen}''' is:

$$S_{\text{gen}}''' = \underbrace{\frac{k}{T^2} (\nabla T)^2}_{\geq 0} + \underbrace{\frac{\mu}{T} \Phi}_{\geq 0} \geq 0 \quad (49)$$

In a two-dimensional convection situation the local entropy generation rate (49) yields:

$$S_{\text{gen}}''' = \frac{k}{T^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T} \left\{ 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right\} \geq 0 \quad (50)$$

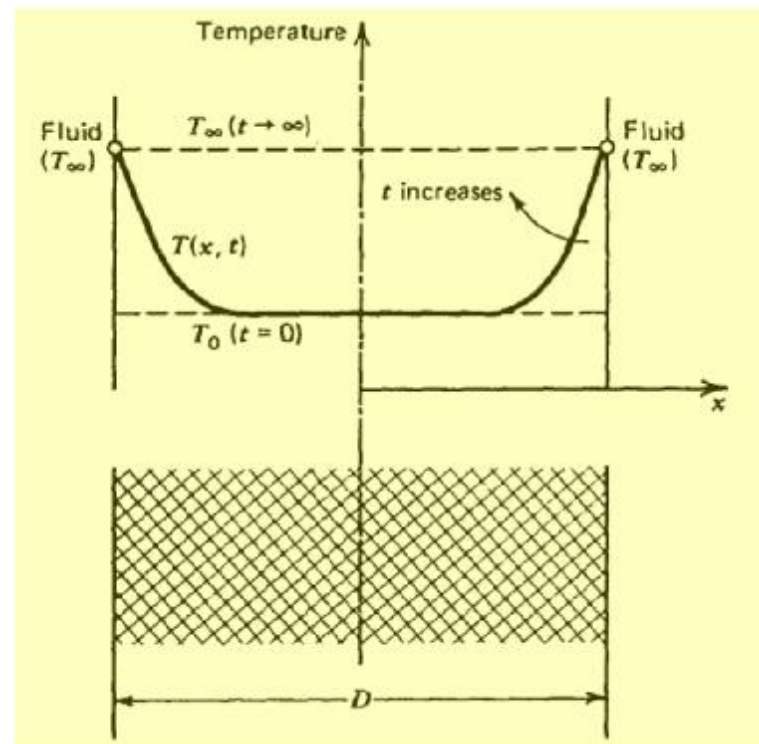
The two-dimensional expression (50) illustrates the competition between viscous dissipation and imperfect thermal contact (finite-temperature gradients) in the generation of entropy via convective heat transfer.

Convection Heat Transfer



RULES OF SCALE ANALYSIS

This section is designed to familiarize the student with the commonsense problem-solving method of scale analysis or scaling.



Transient heat conduction in a slab with sudden temperature change on the boundaries.

Convection Heat Transfer

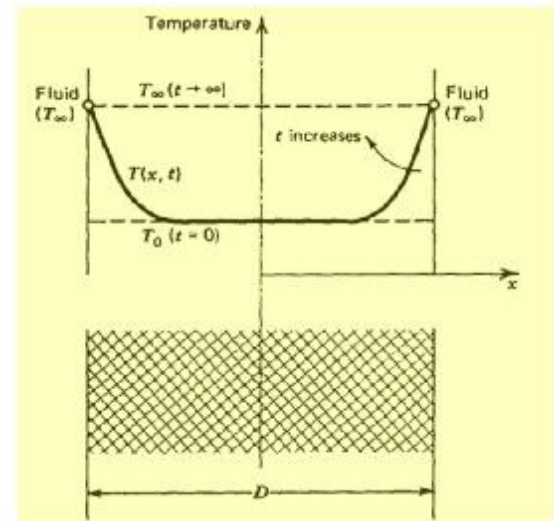


RULES OF SCALE ANALYSIS

We are interested in estimating the time needed by the thermal front to penetrate the plate, that is, the time until the center plane of the plate "feels" the heating imposed on the outer surfaces.

we focus on a half-plate of thickness $D/2$
and the energy equation for pure
conduction in one direction:

$$\rho c_P \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \quad (51)$$



$$\rho c_P \frac{\partial T}{\partial t} \sim \rho c_P \frac{\Delta T}{t}$$

=

$$k \frac{\partial^2 T}{\partial x^2} = k \frac{\partial}{\partial x} \left(\frac{\partial T}{\partial x} \right) \sim \frac{k}{D/2} \frac{\Delta T}{D/2} = \frac{k \Delta T}{(D/2)^2} \quad (52,53)$$

We conclude that:

$$t \sim \frac{(D/2)^2}{\alpha} \quad (54)$$

Convection Heat Transfer



Next session:

✓ Rules of scale analysis

✓ Chapter 2: Laminar Boundary Layer Flow