School of Mechanical Engineering

Sharif University of Technology

Convection Heat Transfer

Instructor: M.H.Saidi

PhD, Professor

Lecture #1

Fall 2011

Fundamentals and Basic Concepts

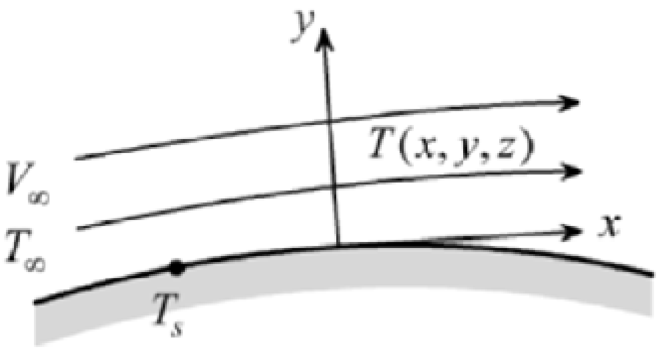
Convection Heat Transfer

Convection heat transfer is the study of heat transport process by fluid

It is not an independent mode of Heat Transfer

It should be represented in terms of conduction or radiation correlations

The objective is to find the convection heat transfer coefficient (h)



$$q''_s = -k \frac{\partial T(x,0,z)}{\partial y}$$

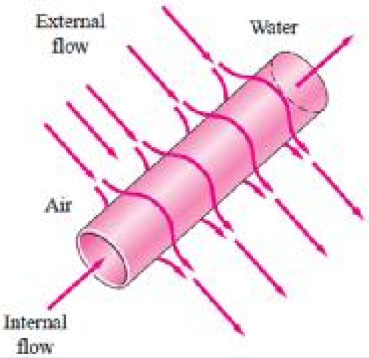
$$q''_s = h(T_s - T_\infty)$$

$$\Longrightarrow \quad h = -k \frac{\frac{\partial T(x,0,z)}{\partial y}}{(T_s - T_\infty)}$$

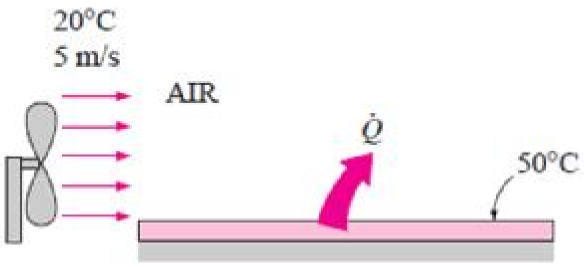
Convection Heat Transfer

Convection heat transfer can be studied in different categories:

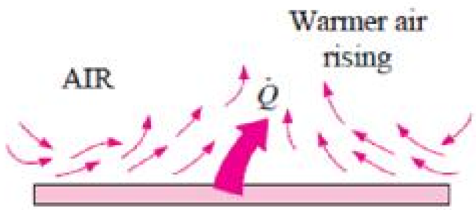
- Forced, free convection or mixed convection
- Internal or external flow
- Laminar or turbulent flow



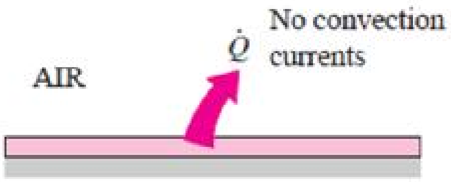
Internal flow of water in a pipe and the external flow of air over the same pipe



(*a*) Forced convection



(*b*) Free convection

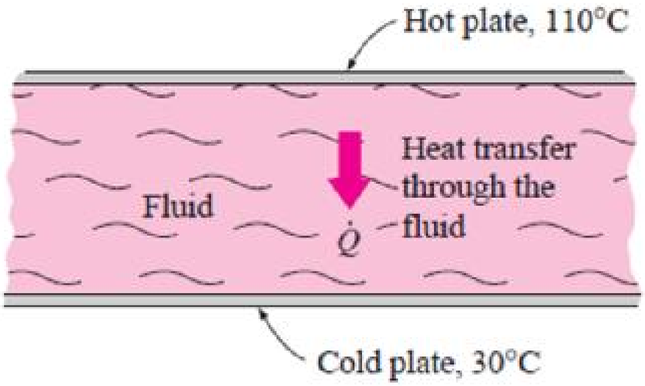


(*c*) Conduction

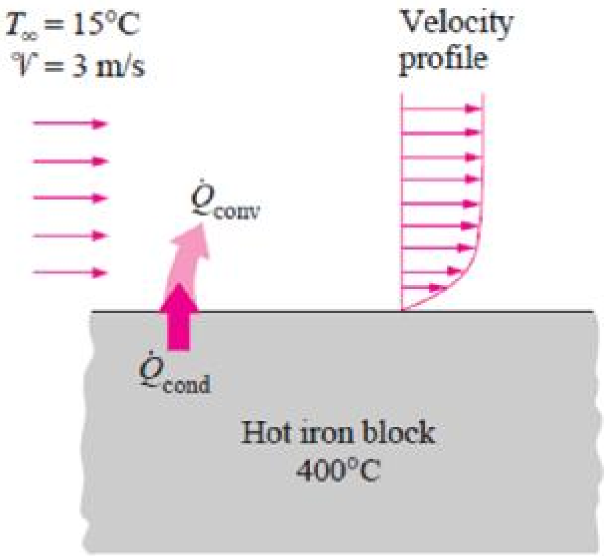
Heat transfer from a hot surface to the surrounding fluid by convection and conduction

Convection Heat Transfer

Typical examples of convection heat transfer



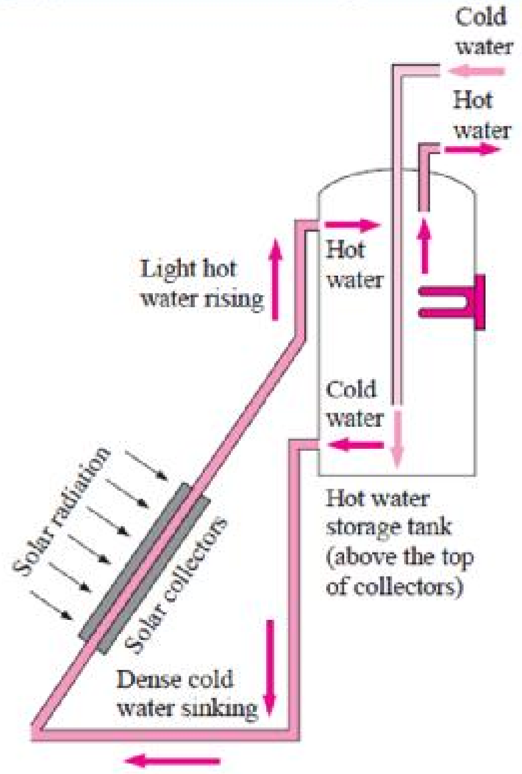
Heat transfer through a fluid sandwiched between two parallel plates



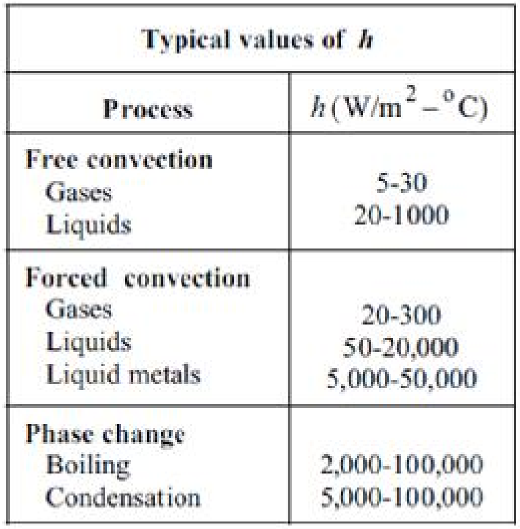
The cooling of a hot block by forced convection

Convection Heat Transfer

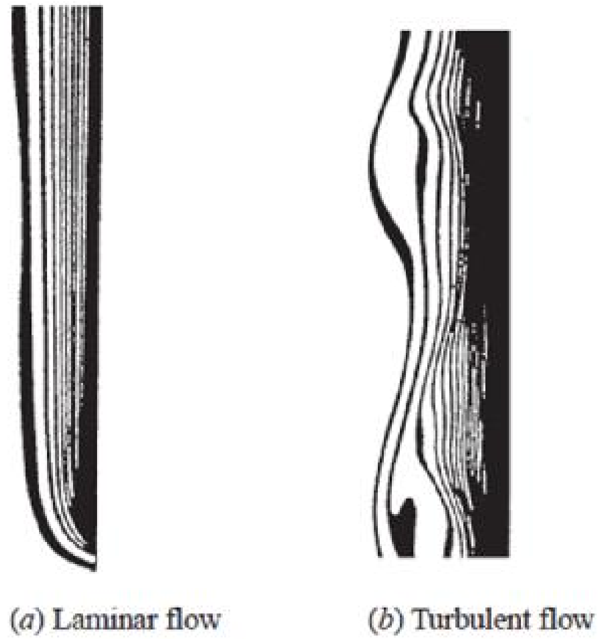
Typical examples of convection heat transfer



Natural circulation of water in a solar water heater by thermosiphoning



Typical values of h	
Process	h (W/m^2 – °C)
Free convection	
Gases	5-30
Liquids	20-1000
Forced convection	
Gases	20-300
Liquids	50-20,000
Liquid metals	5,000-50,000
Phase change	
Boiling	2,000-100,000
Condensation	5,000-100,000



(*a*) Laminar flow

(*b*) Turbulent flow

Isotherms in natural convection over a hot plate in air

Convection Heat Transfer

FUNDAMENTAL EQUATIONS IN CONVECTION HEAT TRANSFER

1- Mass conservation equation

2- Momentum principle

3- First law of Thermodynamics

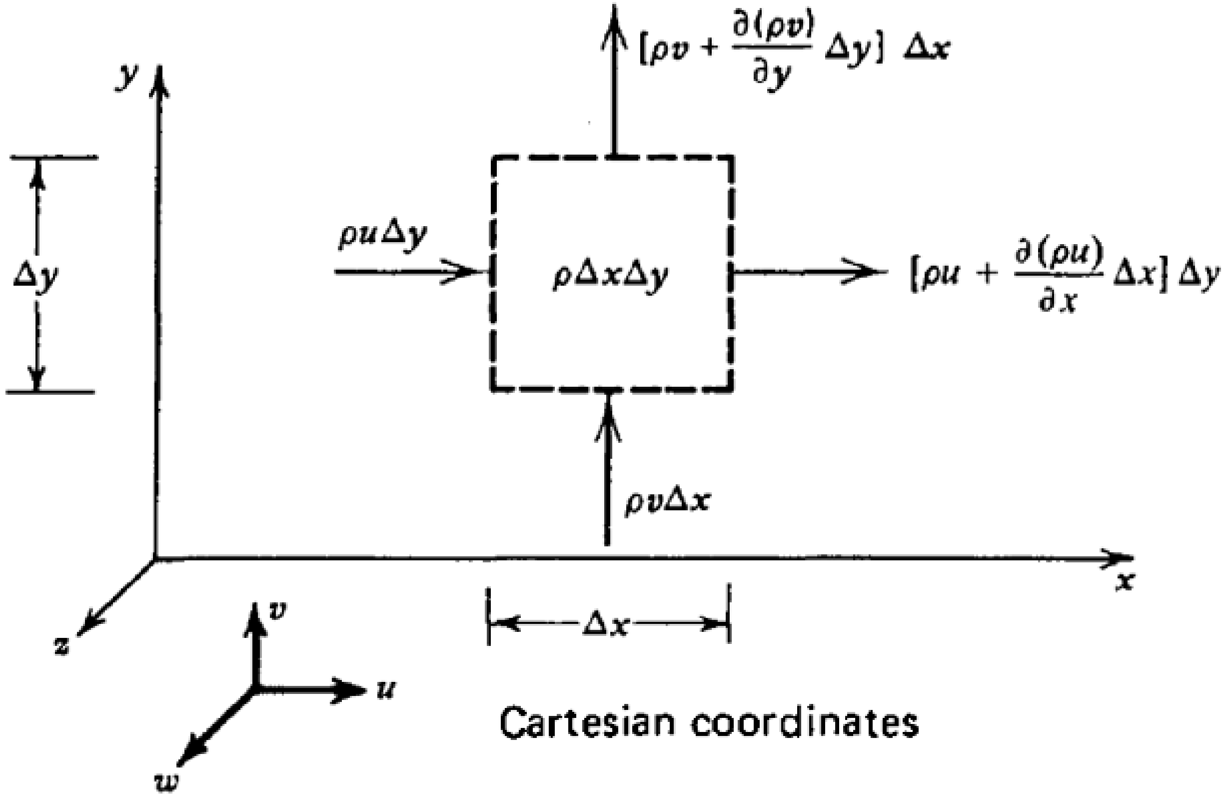
4- Second law of thermodynamics

Convection Heat Transfer

MASS CONSERVATION

q Local approach:Differential form

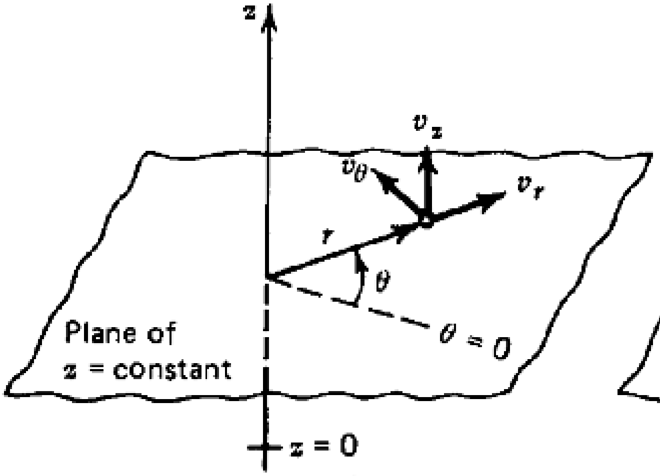
q Integral form: Global approach



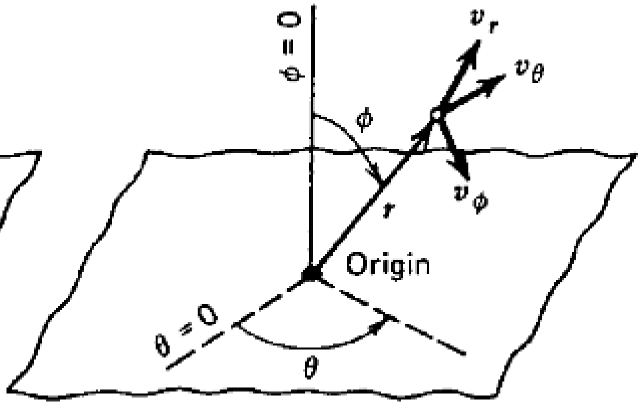
Cartesian coordinates

Convection Heat Transfer

MASS CONSERVATION



(b) Cylindrical coordinates



(c) Spherical coordinates

Convection Heat Transfer

MASS CONSERVATION : Differential Formulation

Dividing through by the constant size of the control volume (**Δ**x **Δ**y),

$$\frac{\partial M_{cv}}{\partial t} = \sum_{\substack{\text{inlet}\\ \text{ports}}} \dot{m} - \sum_{\substack{\text{outlet}\\ \text{ports}}} \dot{m} \qquad (1)$$

Taking u and v as the local velocity components at point {x,y), the mass conservation equation (1) requires that:

$$\frac{\partial}{\partial t}(\rho\,\Delta x\,\Delta y) = \rho u\,\Delta y + \rho v\,\Delta x - \left[\rho u + \frac{\partial(\rho u)}{\partial x}\Delta x\right]\Delta y - \left[\rho v + \frac{\partial(\rho v)}{\partial y}\Delta y\right]\Delta x \qquad (2)$$

Convection Heat Transfer

MASS CONSERVATION : Differential Formulation

It is the conservation of mass in a closed system or the "continuity" of mass through a flow (open) system.

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0 \qquad (3)$$

In a three-dimensional flow, an analogous argument yields

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0 \qquad (4)$$

Convection Heat Transfer

MASS CONSERVATION : Differential Formulation

Expanded formulation of mass conservation equation is:

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \qquad (5)$$

or

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{v} = 0 \qquad (6)$$

D/Dt represents the "material derivative" operator,

$$\frac{D}{Dt} = \underbrace{\frac{\partial}{\partial t}}_{\text{Temporal derivative}} + \underbrace{u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}}_{\text{Spatial derivative}} \qquad (7)$$

If temporal and spatial variations in density are negligible relative to the local variations in velocity, we have:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (8)$$

Convection Heat Transfer

MASS CONSERVATION : Differential Formulation

The equivalent forms of eq. (8) in cylindrical and spherical coordinates:

Cylindrical coordinate

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \qquad (9)$$

Spherical coordinate

$$\frac{1}{r}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{\sin\phi}\frac{\partial}{\partial \phi}(v_\phi \sin\phi) + \frac{1}{\sin\phi}\frac{\partial v_\theta}{\partial \theta} = 0 \qquad (10)$$

These forms are valid only for incompressible fluids;
In fact, their derivation shows that they apply to flows (not fluids) where the density and velocity gradients are such that the D**r**/Dt terms are negligible relative to the **r**∇ • v terms in eq. (6).

Most of the gas flows encountered in heat exchangers, heated enclosures, and porous media obey the simplified version of the mass conservation principle.

Convection Heat Transfer

Assignment #1

Considering the differential approach, derive the mass conservation equations in the cylindrical and spherical coordinate systems

Due date: One week after this session

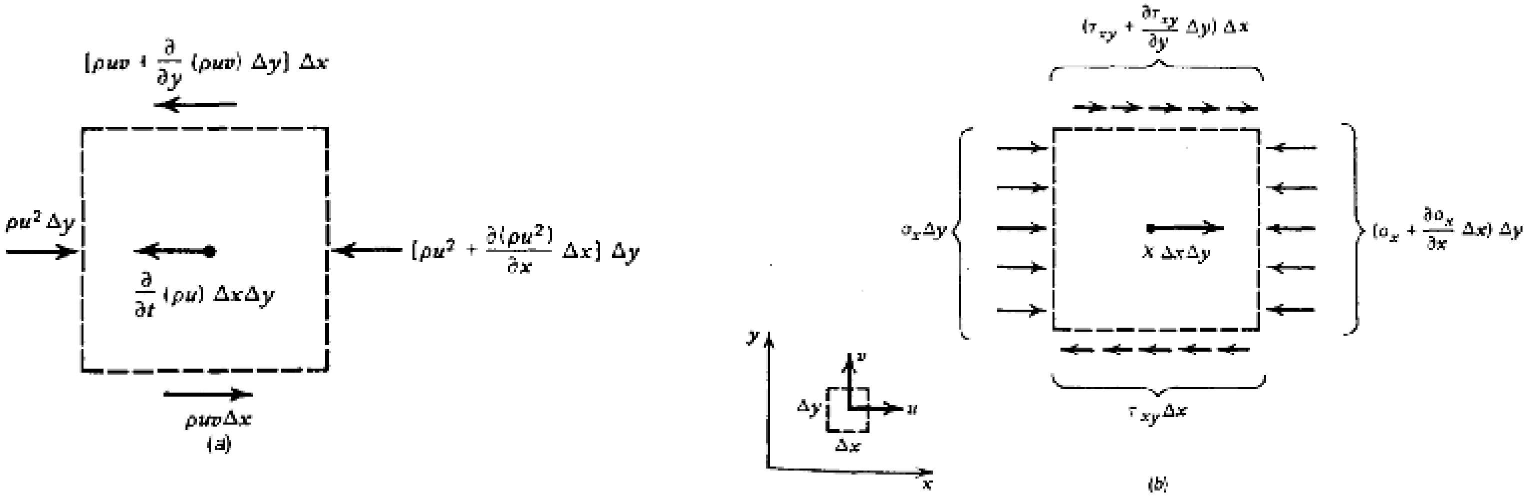
Convection Heat Transfer

MOMENTUM PRINCIPLE : Differential Formulation

The force balance on a control volume requires that:

$$\frac{\partial}{\partial t}(Mv_n)_{cv} = \sum F_n + \sum_{\substack{\text{inlet}\\\text{ports}}} \dot{m}v_n - \sum_{\substack{\text{outlet}\\\text{ports}}} \dot{m}v_n \qquad (11)$$

in which F represents for surface and body forces



Force balance in the x direction on a control volume in two-dimensional flow.

Convection Heat Transfer

MOMENTUM PRINCIPLE : Differential Formulation

Projecting all these forces on the x axis, we obtain:

$$\begin{aligned}
&-\frac{\partial}{\partial t}(\rho u\,\Delta x\,\Delta y) + \rho u^2\,\Delta y - \left[\rho u^2 + \frac{\partial}{\partial x}(\rho u^2)\,\Delta x\right]\Delta y \\
&\quad + \rho uv\,\Delta x - \left[\rho uv + \frac{\partial}{\partial y}(\rho uv)\,\Delta y\right]\Delta x \\
&\quad + \sigma_x\,\Delta y - \left(\sigma_x + \frac{\partial \sigma_x}{\partial x}\,\Delta x\right)\Delta y - \tau_{xy}\,\Delta x \\
&\quad + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y}\,\Delta y\right)\Delta x + X\,\Delta x\,\Delta y = 0
\end{aligned} \qquad (12)$$

Dividing by ΔxΔy Ay in the limit (ΔxΔy) ⟶ 0, we have:

$$\rho\frac{Du}{Dt} + u\left[\frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] = -\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + X \qquad (13)$$

where X is body force in the x direction

Convection Heat Transfer

MOMENTUM PRINCIPLE : Differential Formulation

According to the mass conservation equation (6), the quantity in brackets is equal to zero, as such:

$$\rho\frac{Du}{Dt}+u\left[\frac{D\rho}{Dt}+\rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]=-\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+X \longrightarrow \rho\frac{Du}{Dt}=-\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+X \qquad (14)$$

≡0 ; due to continuity

Next, we relate the stresses σ_x and τ_{xy} to the local flow field by recalling the constitutive relations:

$$\sigma_x=P-2\mu\frac{\partial u}{\partial x}+\frac{2}{3}\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \qquad (15)$$

$$\tau_{xy}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \qquad (16)$$

Convection Heat Transfer

MOMENTUM PRINCIPLE : Differential Formulation

Combining eqs. (14)-(16) yields the Navier-Stokes equation,

$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left[2\mu \frac{\partial u}{\partial x} - \frac{2\mu}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right] + \frac{\partial}{\partial y}\left[\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] + X \tag{17}$$

In the case when the flow may be treated as incompressible and the viscosity may be regarded as constant, the X-equation will be:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + X \tag{18}$$

For a three-dimensional flow

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + X \tag{19}$$

MOMENTUM PRINCIPLE : Differential Formulation

Vector form of the momentum equation:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla P + \mu \nabla^2 \mathbf{v} + \mathbf{F} \tag{20}$$

r-Momentum equation in cylindrical and spherical coordinate systems:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial P}{\partial r} + \mu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right) + F_r \tag{21}$$

$$\rho\left(\frac{Dv_r}{Dt} - \frac{v_\phi^2 + v_\theta^2}{r}\right) = -\frac{\partial P}{\partial r} + \mu\left(\nabla^2 v_r - \frac{2v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\phi}{\partial \phi} - \frac{2v_\phi \cot\phi}{r^2} - \frac{2}{r^2 \sin\phi}\frac{\partial v_\theta}{\partial \theta}\right) + F_r \tag{22}$$

Convection Heat Transfer

MOMENTUM PRINCIPLE : Differential Formulation

Definitions in spherical coordinate system:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r \frac{\partial}{\partial r} + \frac{v_\phi}{r}\frac{\partial}{\partial \phi} + \frac{v_\theta}{r \sin \phi}\frac{\partial}{\partial \theta} \qquad (23)$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin \phi}\frac{\partial}{\partial \phi}\left(\sin \phi \frac{\partial}{\partial \phi}\right) + \frac{1}{r^2 \sin^2 \phi}\frac{\partial^2}{\partial \theta^2} \qquad (24)$$

Convection Heat Transfer

Next session:

- *Thermodynamics laws*
- *An introduction to scale analysis*