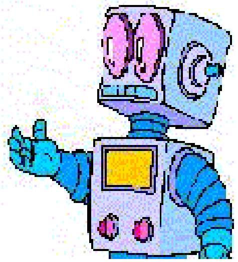
INTRODUCTION TO ROBOTICS

(Kinematics, Dynamics, and Design)

SESSION # 12:

MANIPULATOR

INVERSE KINEMATICS



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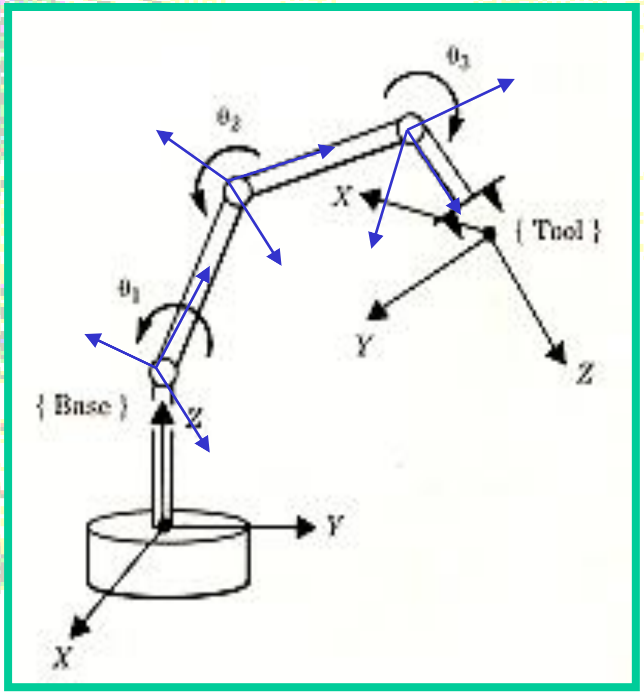
Inverse Manipulator Kinematics

- **Forward Kinematics:** **Describe the position and orientation of the manipulator's end-effector as a function of joint variables relative to a base frame.**

- **Inverse Kinematics:** **Given the desired position and orientation of the end-effector relative to the base, compute the set of joint variables which will achieve this desired result.**



A 3-DOF Manipulator Arm

Inverse Kinematics

Inverse Manipulator Kinematics

- **Solvability (قابل حل بودن):**

Solving kinematics equations in robotics is a Non-Linear Problem.

Given; ${}^{0}_{n}T$ **, Find;** $\{\theta_1, \theta_2, \ldots, \theta_n\}$**, is a** ***non-linear*** **problem.**

Ex: PUMA-560 Robot. Given; ${}^{0}_{6}T$ **, Find;** $\{\theta_1, \theta_2, \ldots, \theta_6\}$**?**

$$
\begin{aligned}
r_{11} &= C_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] + S_1(S_4C_5C_6 + C_4S_6),\\
r_{21} &= S_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] - C_1(S_4C_5C_6 + C_4S_6),\\
r_{31} &= -S_{23}(C_4C_5C_6 - S_4S_6) - C_{23}S_5C_6,\\
&\ldots\\
&\ldots \qquad Equation:(3.14)
\end{aligned}
$$

Inverse Manipulator Kinematics

- **Solvability (قابل حل بودن):**

For a 6-DOF manipulator, we have:

$$ {}^{0}_{6}T = \left[\begin{array}{ccc|c} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

- **12-Equations, and 6-Unknowns?**
- **From 9-Equations of the Rotation Matrix, only 3-Equations are independent.**
- **Therefore, we have 6-independent non-linear equations and 6-unknowns.**

Inverse Manipulator Kinematics

A manipulator is *solvable* if the joint variables can be determined by an algorithm. This algorithm should find all possible solutions.

We have 6-independent non-linear equations and 6-unknowns. Therefore, we should investigate the followings:

- **Existence of Solution (وجود جواب) .**
- **Multiple Solutions (تعدد جواب ها) .**
- **Method of Solution (روش حل) .**

Inverse Manipulator Kinematics

- **Existence of Solution(وجود جواب) :**

Existence of solution to Inv.-Kin. problem depends on the existence of the specified goal point in the manipulator's Workspace.

Workspace/Work-envelope(فضای کاری) : is that volume of space which the end-effector of a robot can reach.

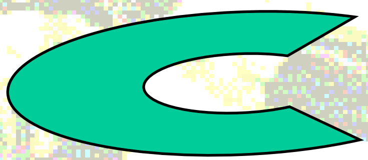
Dexterous Workspace(فضای کاری ماهر) : is that volume of space which the end-effector of a robot can reach with all orientations.

Inverse Manipulator Kinematics

- **Solvability (قابل حل بودن):**

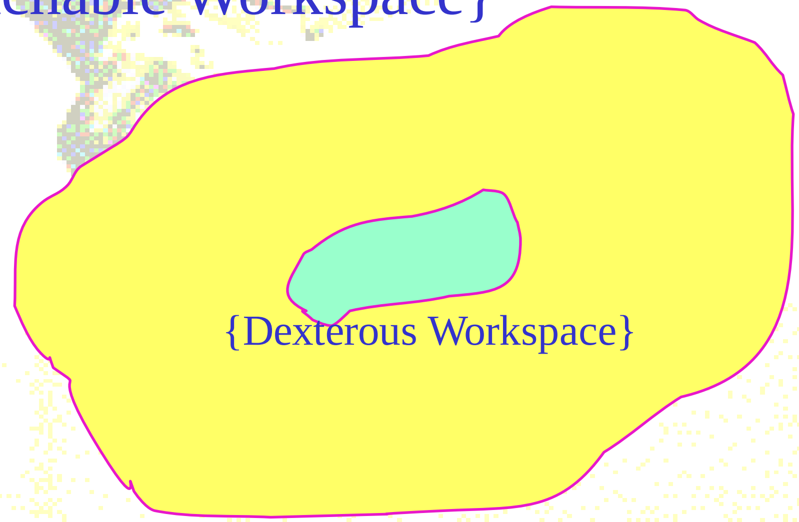
Reachable Workspace(فضای کاری قابل دسترس) : is that volume of space which the end-effector of a robot can reach with at least one orientations.

{Dexterous Workspace}



{Reachable Workspace}

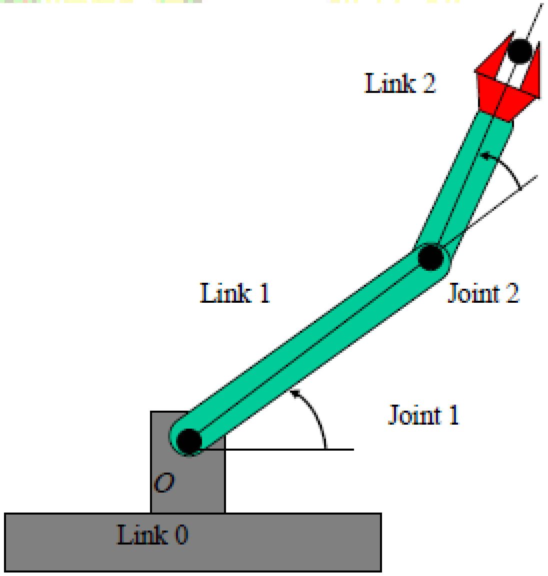
{Reachable Workspace}



Inverse Manipulator Kinematics

- **Workspace (فضای کاری قابل دسترس):**

To describe Manipulator's Work Space: Compute direct kinematics, vary joint variables within their range of motion, and then sketch the results.



Inverse Manipulator Kinematics

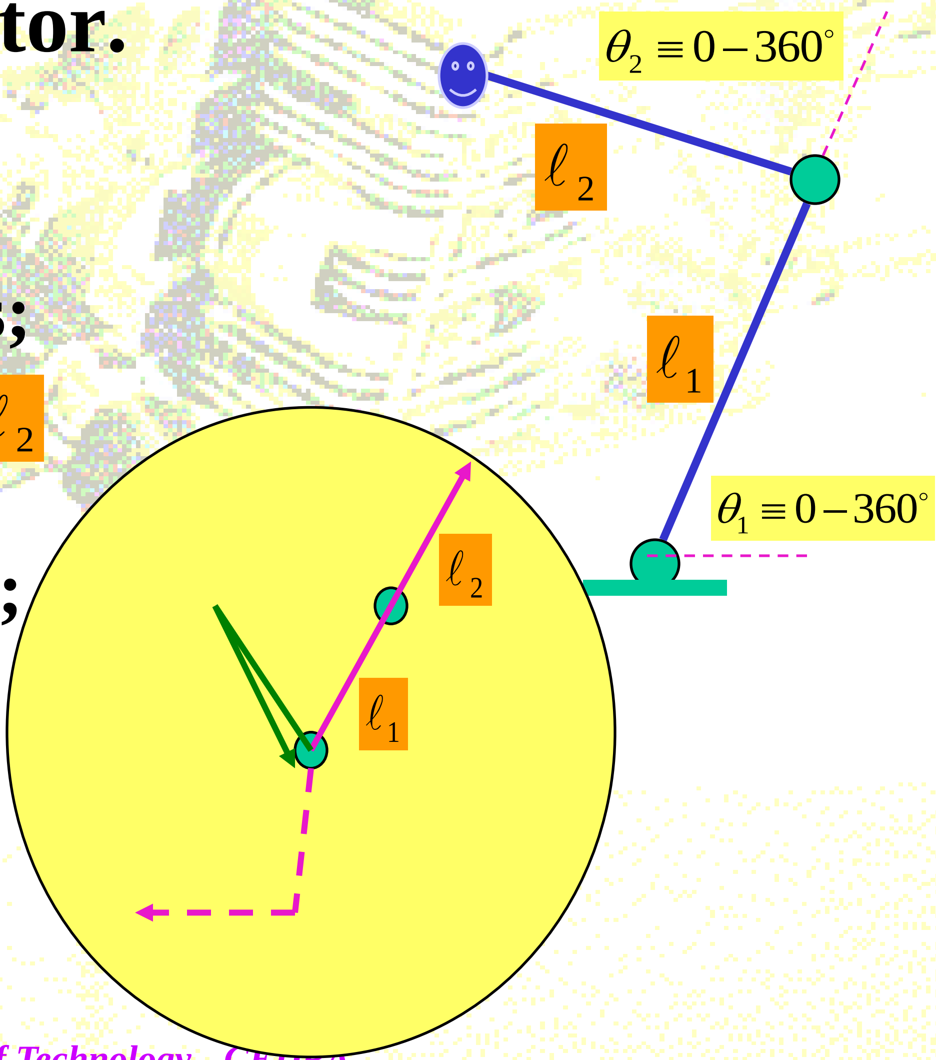
- **Example:** **Consider the workspace of a two-link planar manipulator.**
  - **If $\ell_1 = \ell_2$, then:**

Reachable Workspace is;

Disk of Radius = $2\ell_1 = 2\ell_2$

Dexterous Workspace is;

Origin.



Inverse Manipulator Kinematics

- **Example:** **Consider the workspace of a two-link planar manipulator.**

If $\ell_1 \neq \ell_2$, **and** $\ell_1 > \ell_2$ **then:**

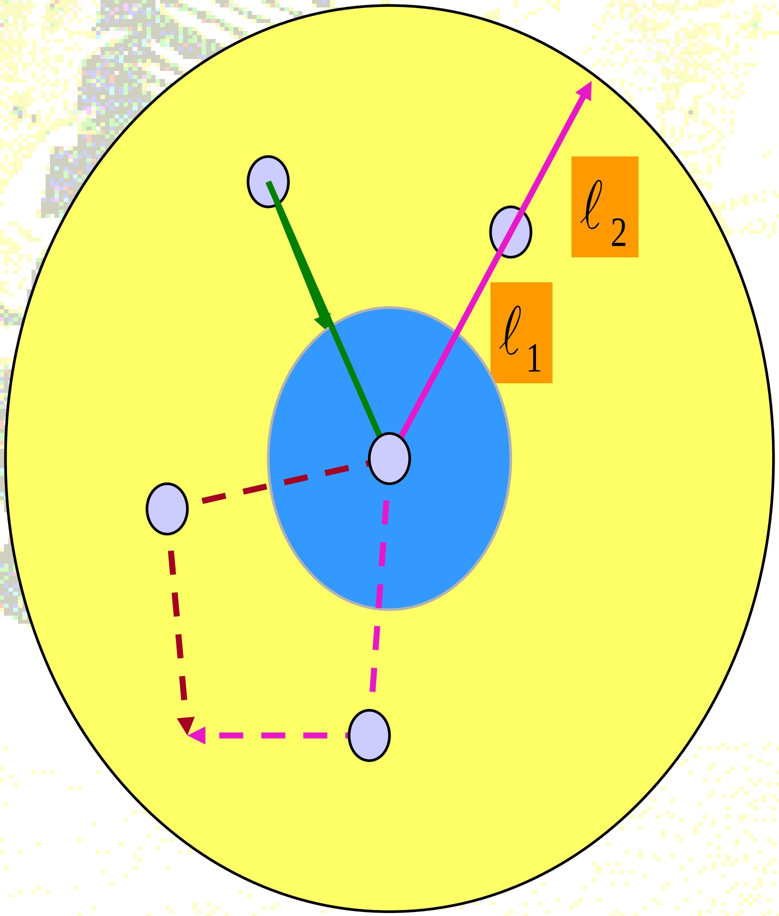
Reachable Workspace is;

A Disk as shown:

$$Outer - Radius \equiv \ell_1 + \ell_2$$

$$Inner - Radius \equiv \left|\ell_1 - \ell_2\right|$$

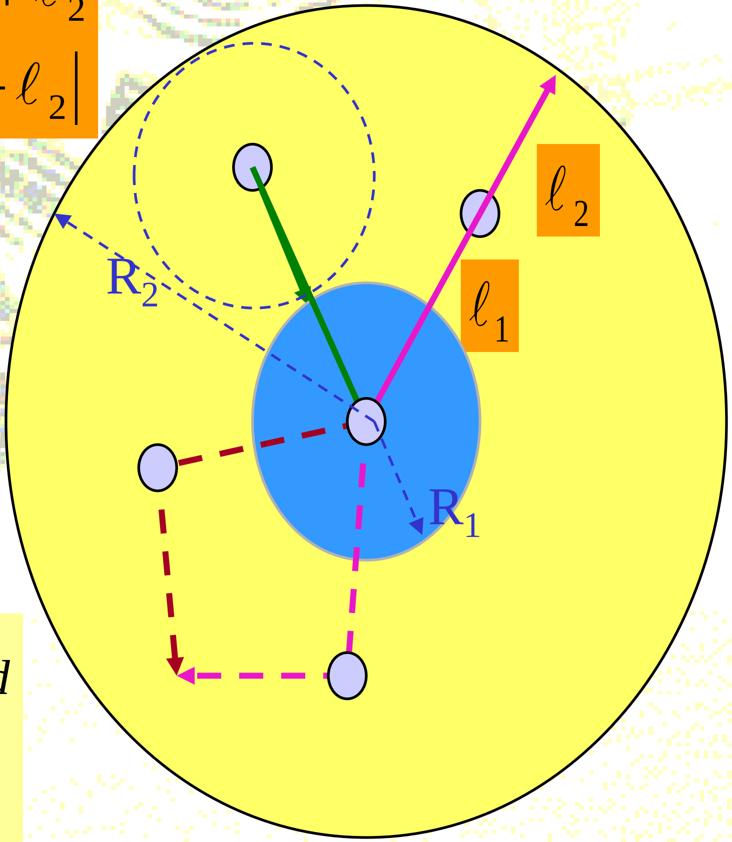
Dexterous Workspace does not exist.



Inverse Manipulator Kinematics

- **Example:** **For the end-effector of this robot to reach all points within its workspace, what should be the links lengths?**

$$OR \equiv R_2 = \ell_1 + \ell_2$$
$$IR \equiv R_1 = |\ell_1 - \ell_2|$$



- **If** $\ell_1 > \ell_2$ **then:**

$$R_1 + R_2 = (\ell_1 - \ell_2) + (\ell_1 + \ell_2) \Rightarrow \ell_1 = \frac{R_1 + R_2}{2}, \quad and$$
$$\ell_2 = R_2 - \ell_1 = R_2 - \frac{R_1 + R_2}{2} = \frac{R_2 - R_1}{2}$$

- **If** $\ell_1 < \ell_2$ **then:**

$$R_1 + R_2 = -(\ell_1 - \ell_2) + (\ell_1 + \ell_2) \Rightarrow \ell_2 = \frac{R_1 + R_2}{2}, \quad and$$
$$\ell_1 = R_2 - \ell_2 = R_2 - \frac{R_1 + R_2}{2} = \frac{R_2 - R_1}{2}$$

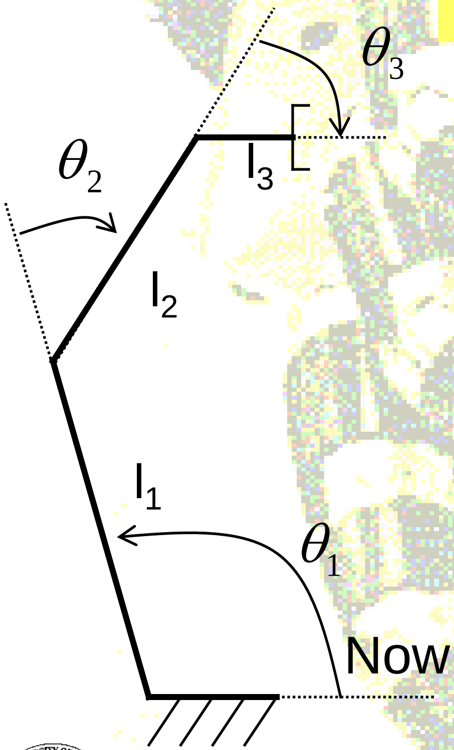
Example: Planar 3-link robot

$$x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) + l_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) + l_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$\phi = \theta_1 + \theta_2 + \theta_3$$

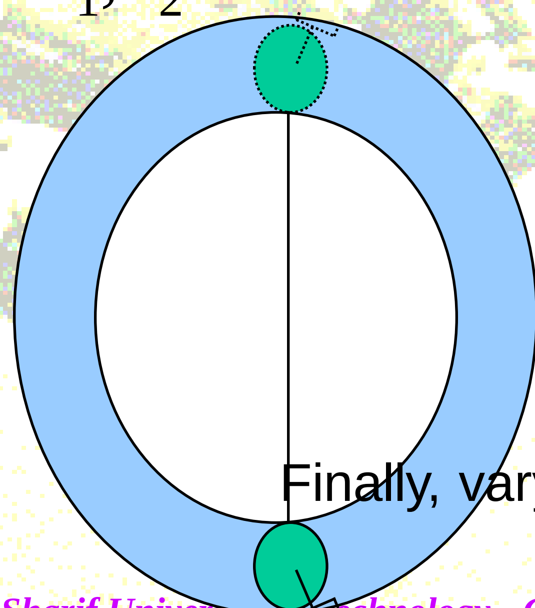
$$\text{Take } l_1 > l_2 > l_3,\ l_1 > l_2 + l_3$$



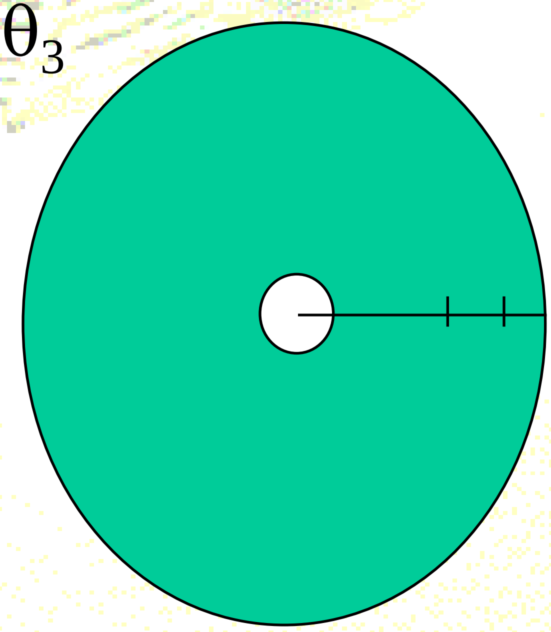
What is the reachable workspace?

Take l_1, l_2 fixed and vary θ_3

Now vary θ_1



Finally, vary θ_2



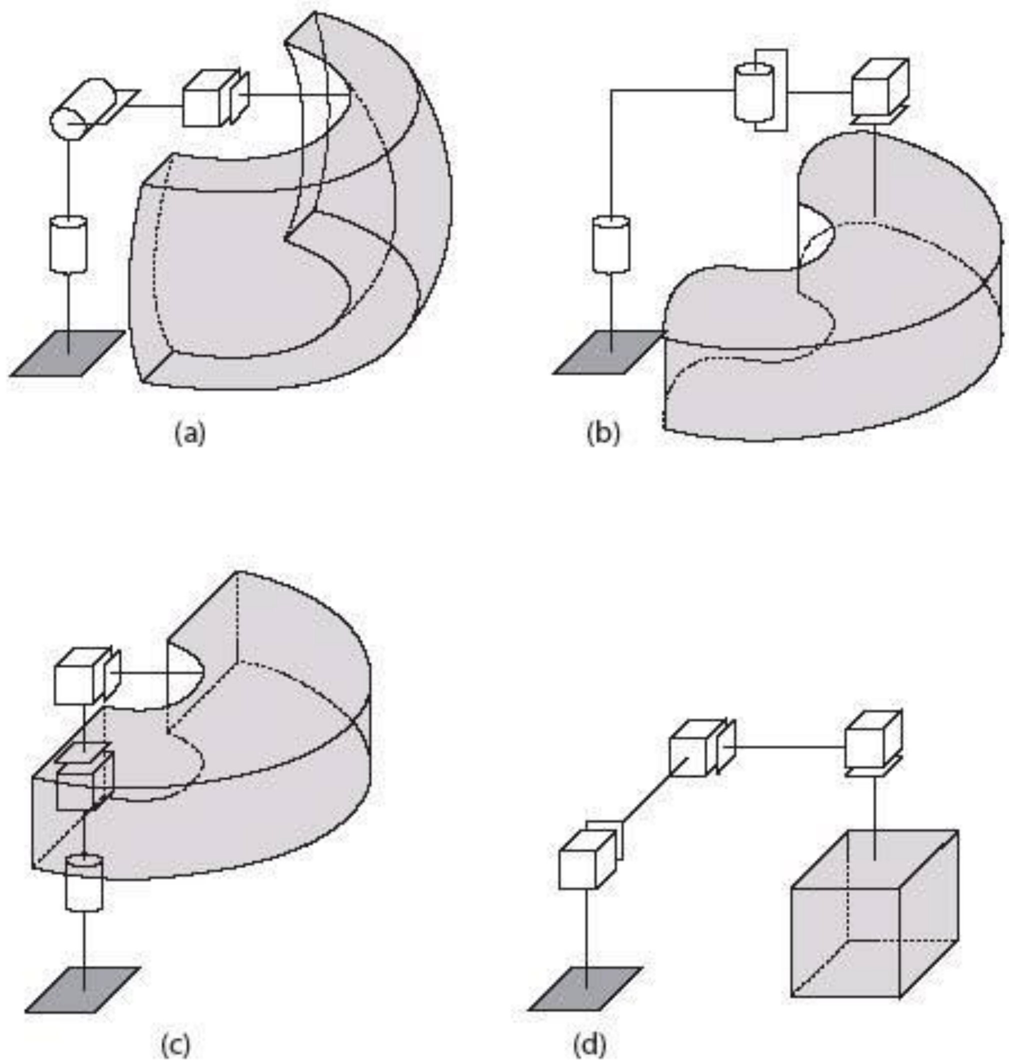
Workspace Comparison

(a) Spherical (RRP)

(b) SCARA (RRP)

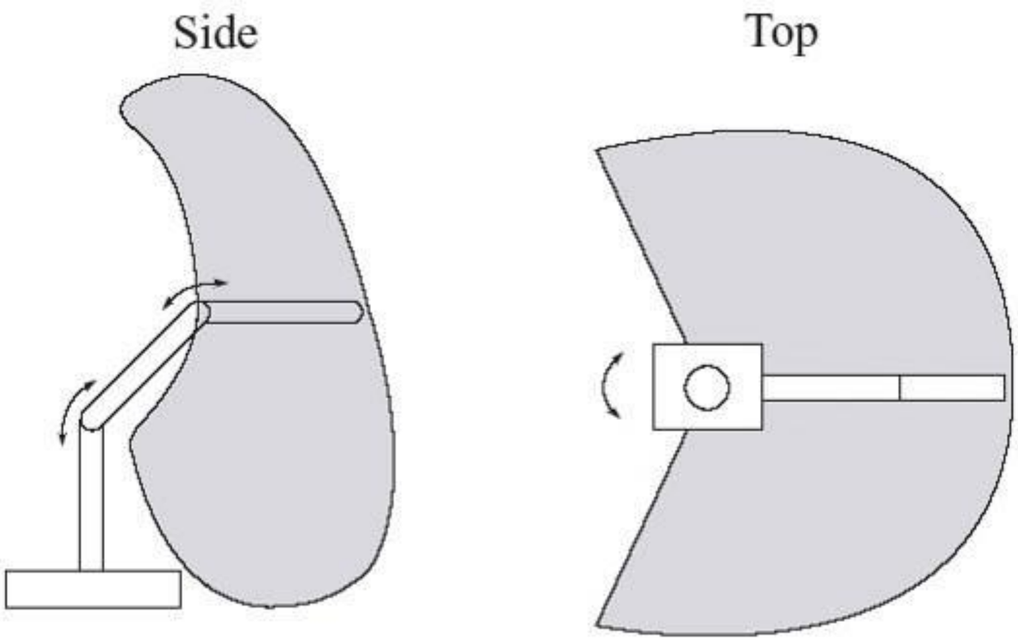
(c) Cylindrical (RPP)

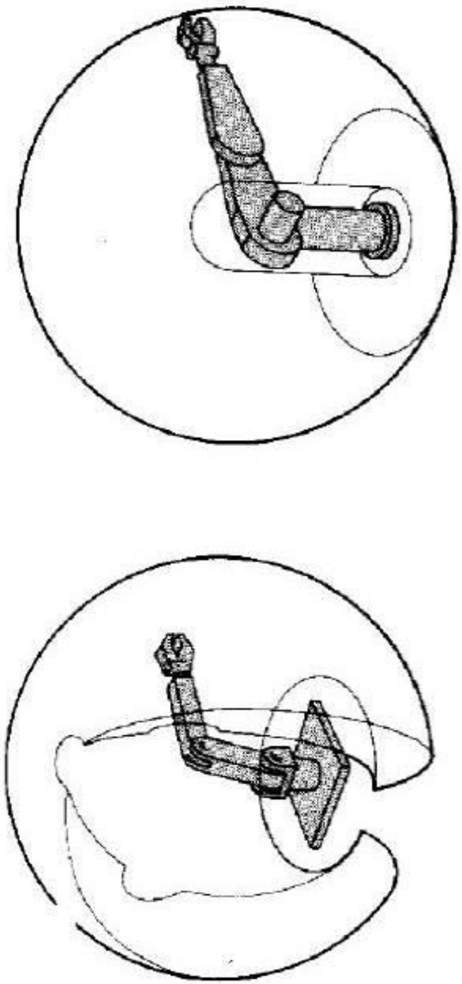
(d) Cartesian (3P)



Workspace Comparison

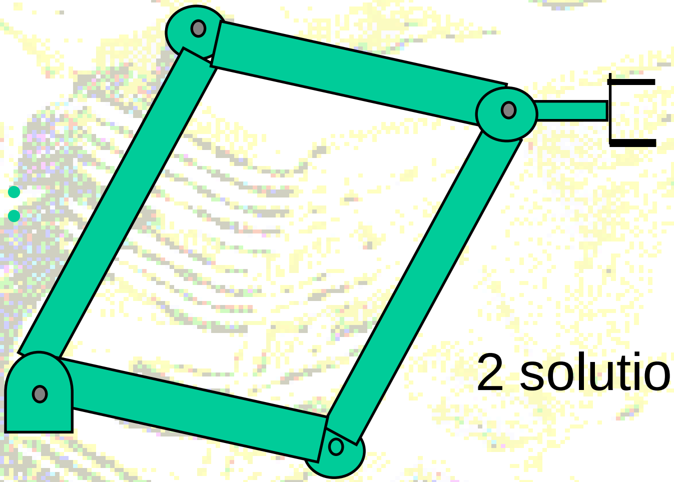
A 3R Elbow Manipulator





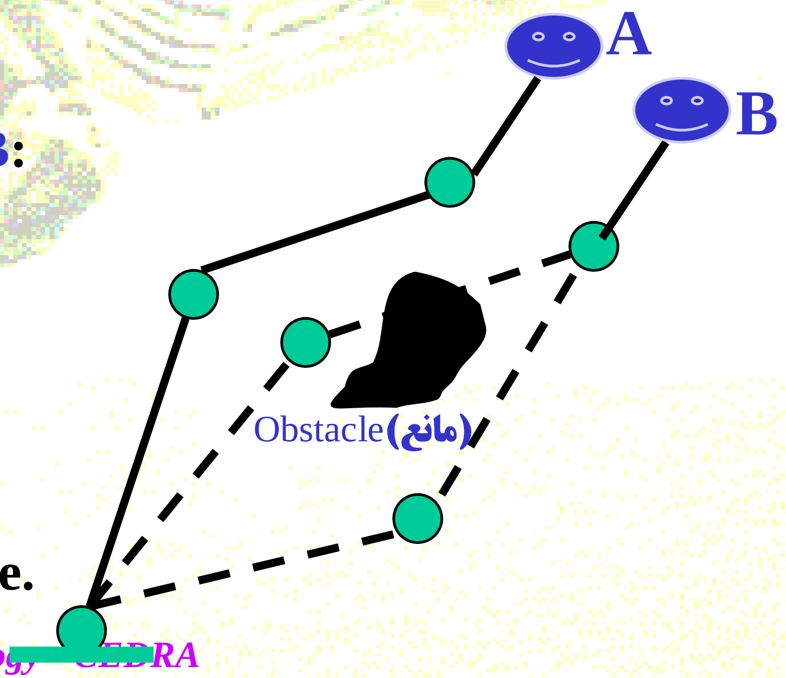
Inverse Manipulator Kinematics

- **Solvability (قابل حل بودن):**
  - **Multiple Solutions(تعدد جوابها) :**



A manipulator may reach any position in the interior of its workspace with different configurations. But the system has to be able to choose one.

A manipulator moving from point A to B:



Two solutions exist:

- One causes a collision, and

- Other is safe.

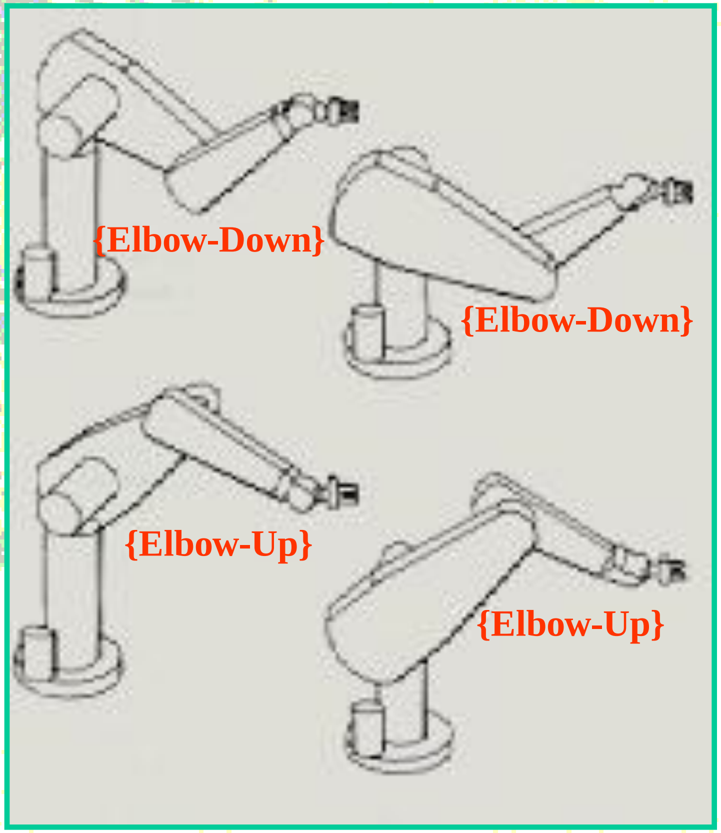
Therefore, we need to find all solutions, and teach the robot to choose the best one.

Inverse Manipulator Kinematics

- **Solvability (قابل حل بودن):**

➢ **Multiple Solutions(تعدد جوابها) :**

Ex: The PUMA-560 manipulator can reach certain goals with 8-different solutions. Due to the limits imposed on joints ranges, some of these solutions may not be accessible.



{Other 4-solutions are for the wrist}

Inverse Manipulator Kinematics

- **Solvability (قابل حل بودن):**

➢ **Multiple Solutions(تعدد جوابها) :**

In general number of solutions depends on the non-zero D-H link parameters, and the allowable rang of motion for the joints.

- The more Non-Zero link parameters → The more Solutions to reach a certain goal.

- Up to 16-solutions are possible for a completely general revolute arm with 6-DOF (see table 4.5).

Inverse Manipulator Kinematics

➢Methods of Solutions

A manipulator is *solvable* if the joint variables can be determined by an algorithm. This algorithm should find all possible solutions.

- ❖ **Closed-form solutions**
- ❖ **Numerical solutions**

We are interested in closed-form solutions:

1. **Algebraic Methods**
2. **Geometric Methods**