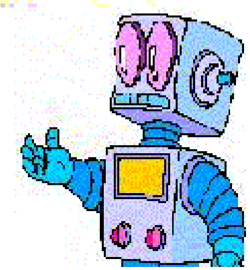
INTRODUCTION TO ROBOTICS

(Kinematics, Dynamics, and Design)

SESSION # 11:

MANIPULATOR KINEMATICS



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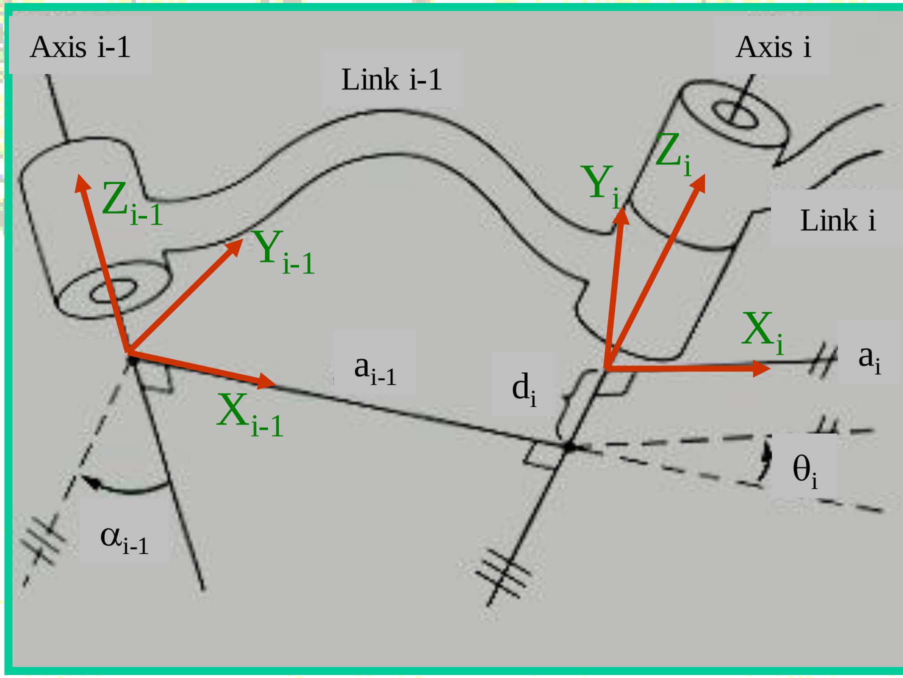
Manipulator Kinematics

- **Affixing Frames to Links (قرارداد اتصال دستگاه به رابطها):**

➢ **To describe relative location of each link to its neighboring link, we shall attach a set of frames to each link in a manipulator in accordance to the following convention (frame {i} is rigidly attached to the link-i):**

❖ Intermediate Links (رابطهای میانی):

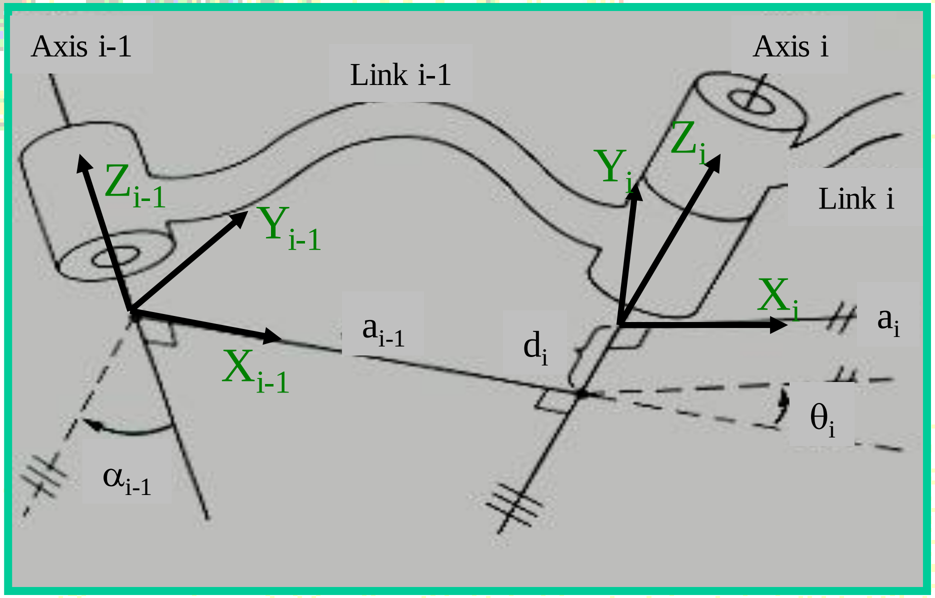
1. **The Z_i-axis of frame-{i}, called "Z_i", is coincident with the joint axis-i.**
2. **The origin of frame-{i} is located where the a_i-perpendicular intersects the "i-th" axis.**
3. **X_i-axis points along "a_i" in the direction from joint "i" to joint "i+1".**
4. **Y_i-axis is formed by the RHR to complete the "i-th" frame.**
5. **If the joint axes intersect, a_i=0, then X_i-axis is chosen normal to the plane of Z_i and Z_{i+1}.** $(\hat{X}_i = \pm(\hat{Z}_i \times \hat{Z}_{i+1}))$



Manipulator Kinematics

- **The "T" Transformation (ماتریس تبدیل–تی):**
  - **We shall now derive the *General form of Transformations* which relates frames attached to neighboring links.**
  - **In general, two neighboring links may be shown as follows:**
  - **We wish to determine the transformation which defines frame {i} relative to the frame {i-1}.**

$$ {}^{i-1}_{i}T = ? \equiv f(a_{i-1}, \alpha_{i-1}, d_i, \theta_i) $$

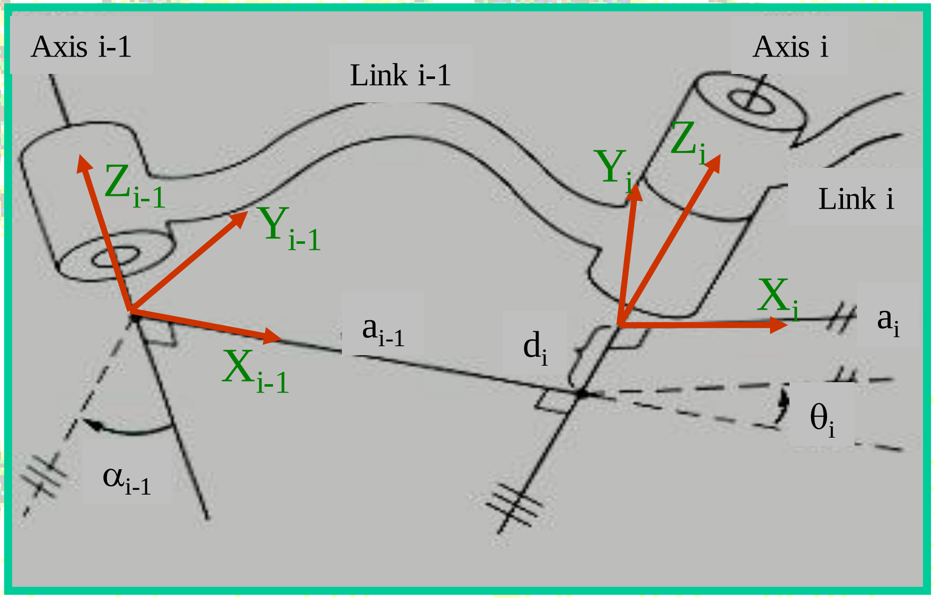


  - **One can easily align frame {i-1}on frame {i} by 4-simple transformations as follows:**

Manipulator Kinematics

- **The "T" Transformation (ماتریس تبدیل–تی):**

1. **Rotate frame {i-1} about X_{i-1} axis by α_{i-1} to make the Z_{i-1} in the same direction as Z_i. $Rot(X_{i-1}, \alpha_{i-1})$**
2. **Translate along X_{i-1} axis by a_{i-1} to bring the two origins on the same axis Z_i. $Trans(X_{i-1}, a_{i-1})$**
3. **Rotate about Z_i axis by θ_i to make X_{i-1} in the same direction as X_i. $Rot(Z_i, \theta_i)$**
4. **Translate along Z_i axis by d_i to make the two frames completely coincide. $Trans(Z_i, d_i)$**



Manipulator Kinematics

- The “T” Transformation (ماتریس تبدیل–تی):

Combining all transformations results in:

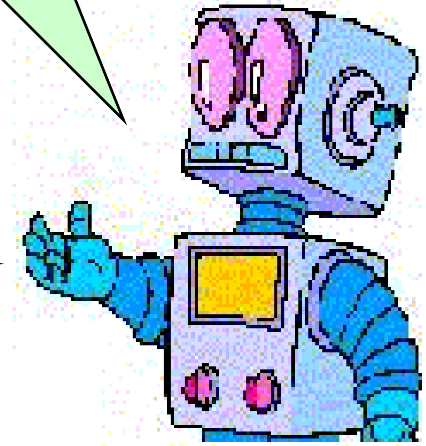
$$ {}^{i-1}_{i}T = Rot(\hat{X}_{i-1},\alpha_{i-1})Trans(\hat{X}_{i-1},a_{i-1})Rot(\hat{Z}_i,\theta_i)Trans(\hat{Z}_i,d_i) \equiv $$

$$ \equiv \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -S\alpha_{i-1} d_i \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & C\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Direct/Forward Kinematics

Where is my hand?

Direct Kinematics:
HERE!



Manipulator Kinematics

- **Forward Kinematics (سینماتیک مستقیم):**

Given the joint variables (θ_1, θ_2, …), compute the Position and Orientation of the last link of the manipulator arm relative to the base frame?

Given:

$$ {}^{i-1}_{i}T, \qquad i = 1, \dots, n $$

$$ {}^{0}_{n}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T \, \dots\dots \, {}^{n-1}_{n}T $$

Where: ${}^{0}_{n}T$ **is function of n joint variables, and represents the Cartesian position & orientation of the last link relative to base frame.**

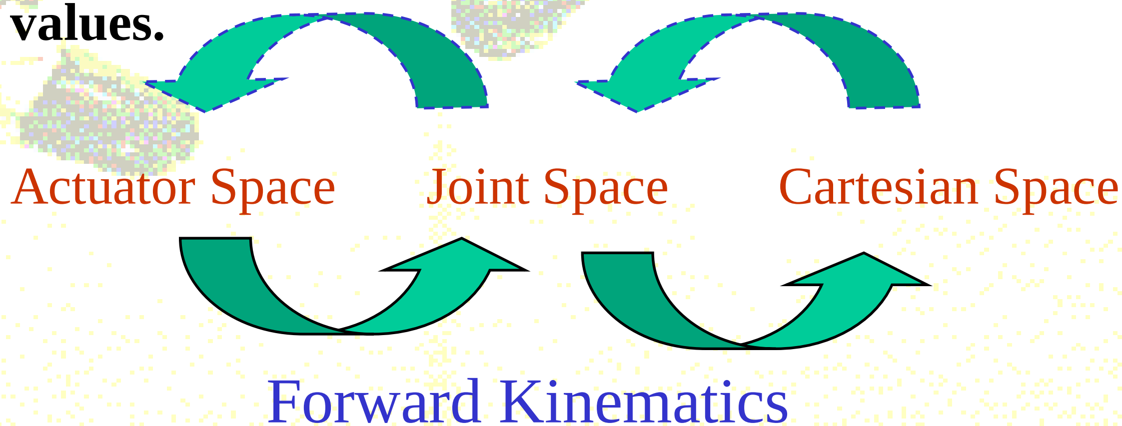
Manipulator Kinematics

- ## Actuator Space, Joint Space, and Cartesian Space:

Joint Space: **Set of joint variables "θ_1, θ_2, …" (the n×1 joint vector) can be used to describe the position of all links of a manipulator.**

Cartesian Space: **Description of position and orientation of the manipulator is done along orthogonal axes using joint space description.**

Actuator Space: **Sometimes a linear actuator is used to rotate a revolute joint using a 4-bar linkage. Since the sensors which measure the position of the manipulator are often located at the actuators, some computations must be performed to compute the joint vector as a function of a set of actuator values.**



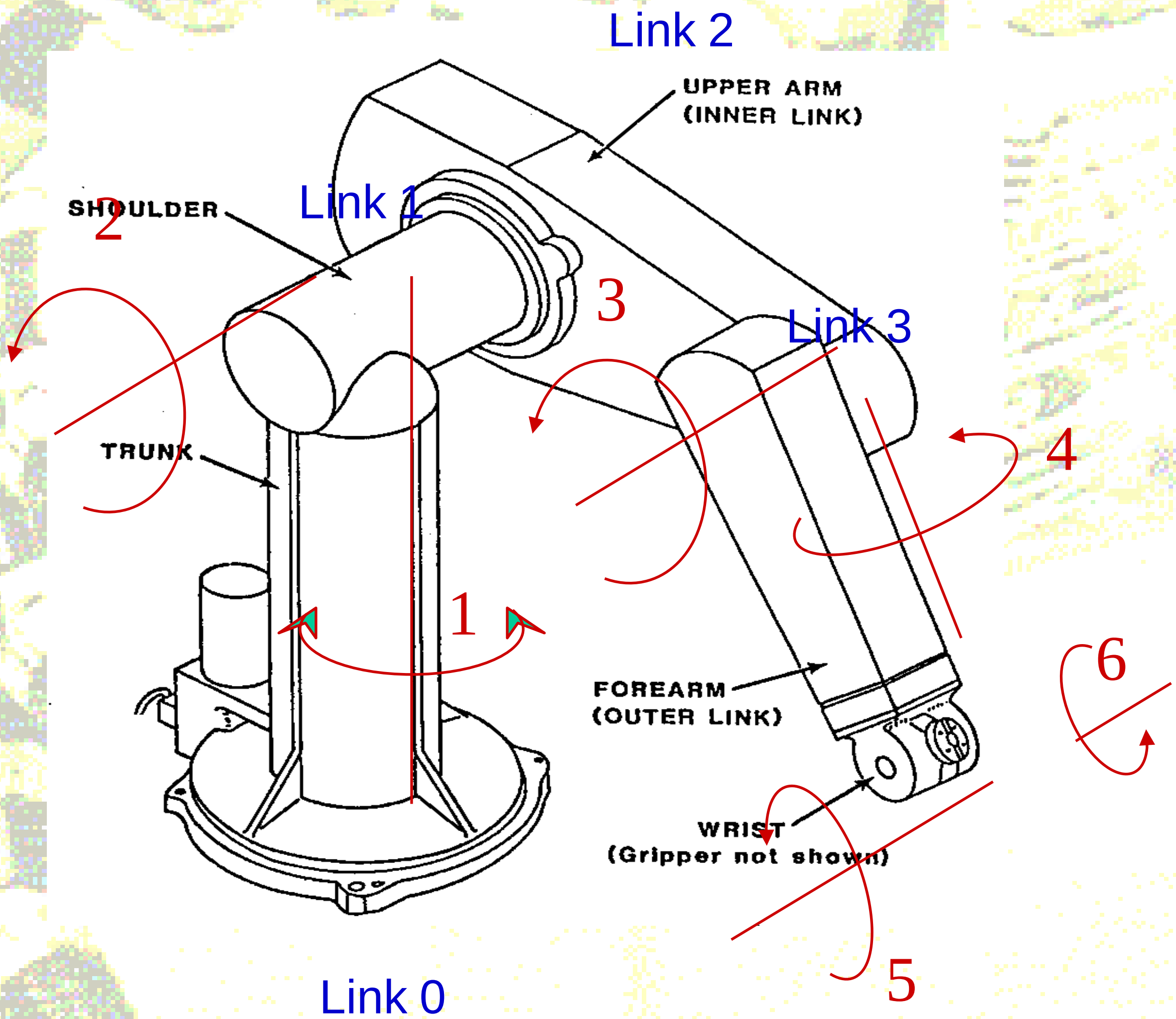
Manipulator Kinematics

- **Example: The Unimation PUMA-560 Robot.**

  - **A 6-DOF Revolute Robot**

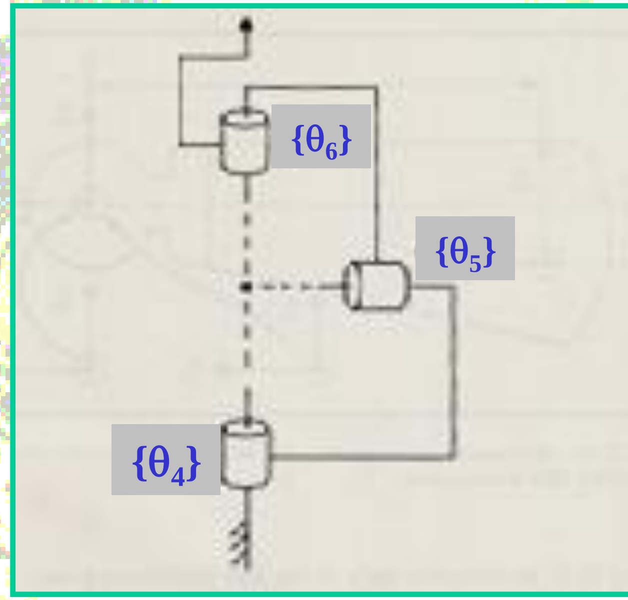
  - **A 6R Robot Mechanism**

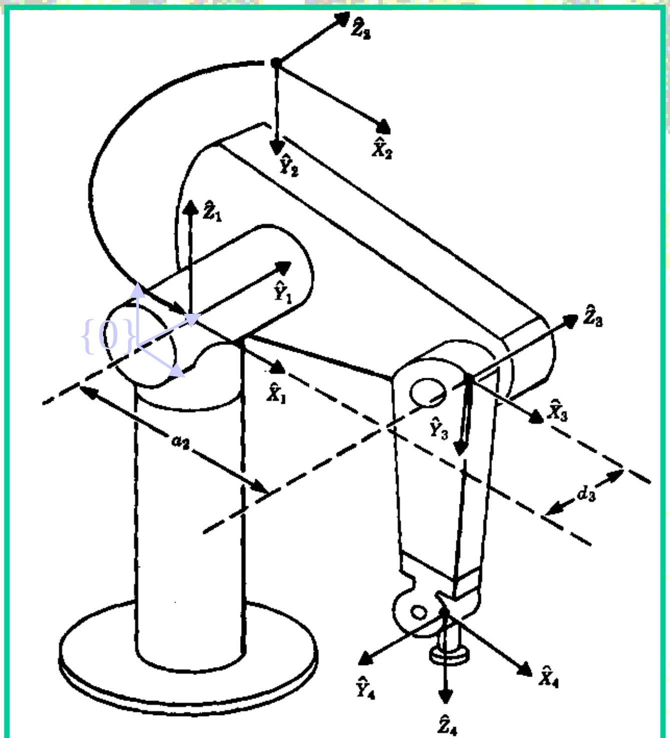
Kinematic Modeling

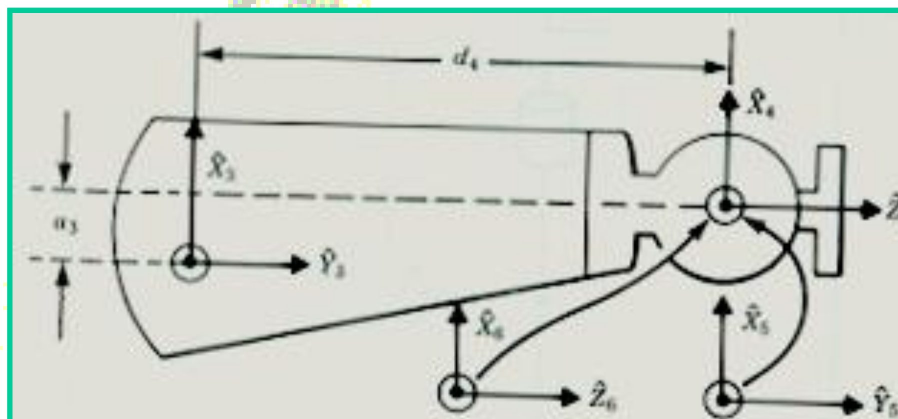


PUMA-560 Manipulator Kinematics

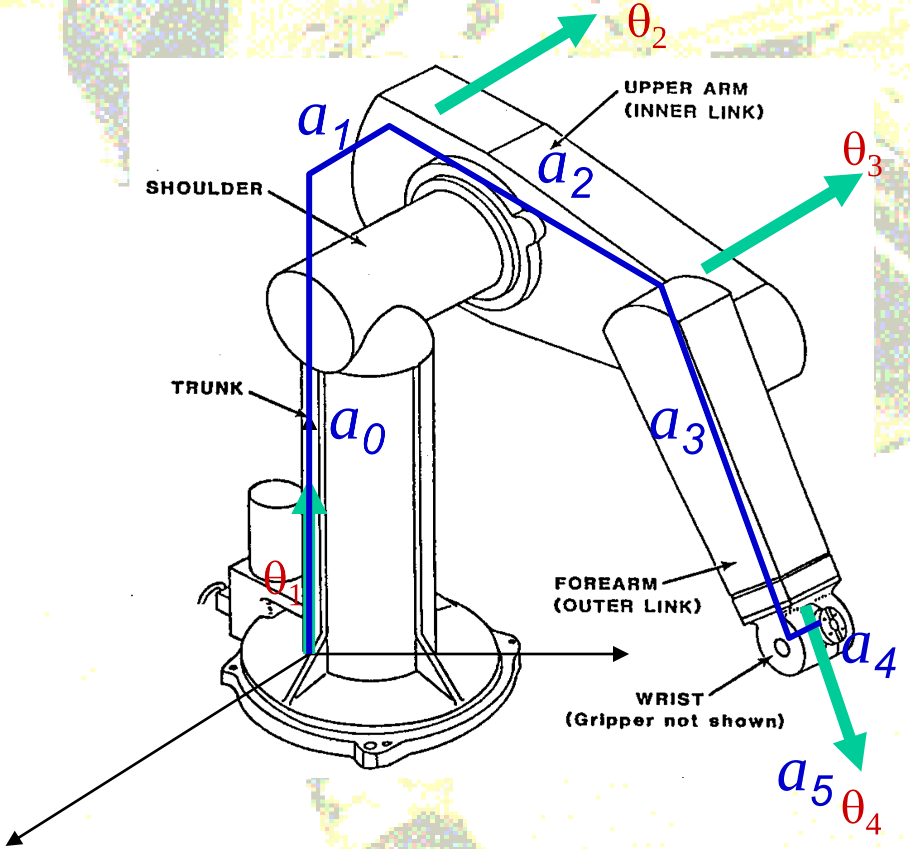
- **Frames Attachment (اتصال چهارچوبها):**
  - **Frame {0} is coincident with frame {1}.**
  - **Joint axes of joints 4, 5, and 6 all intersect at a common point.**

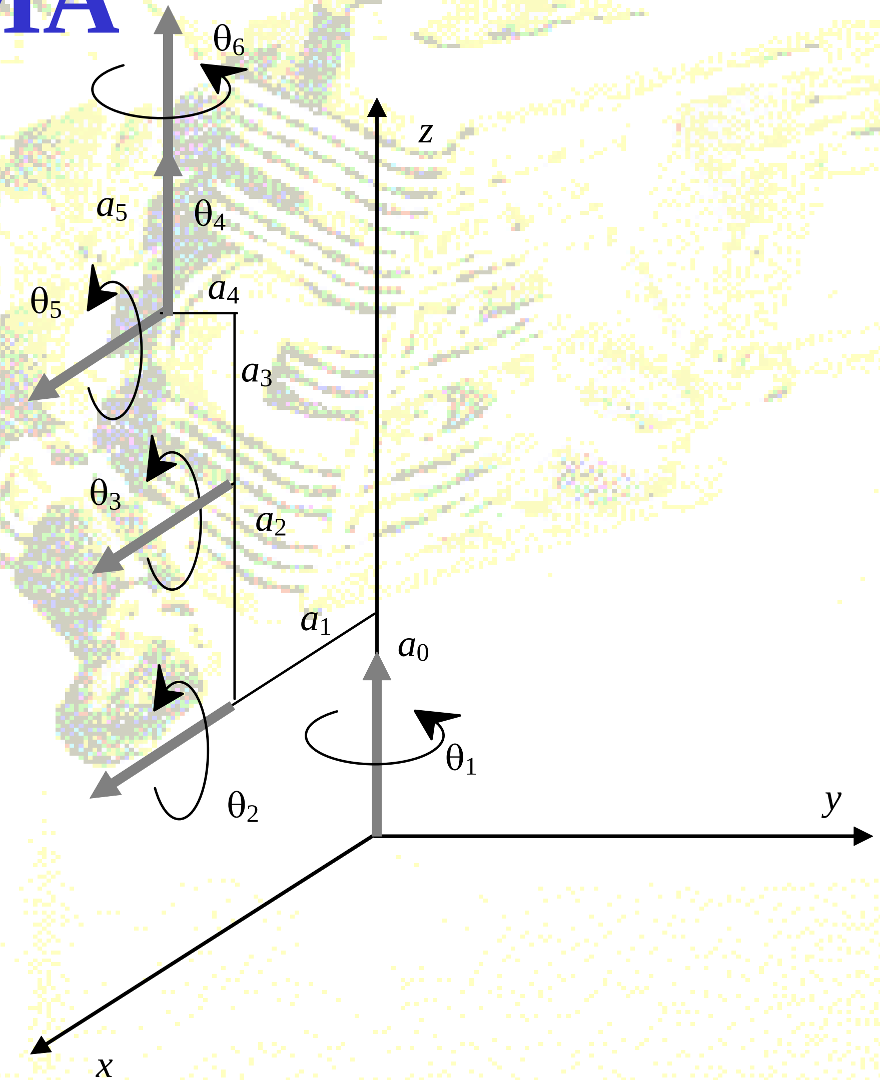






Offsets and Home Position for the PUMA





Manipulator Kinematics

The PUMA-560 Table of Link-Joint Parameters:

Joint-i	${}^{i-1}_{i}T$	θ_i	α_{i-1}	a_{i-1}	d_i
1	${}^{0}_{1}T$	θ_1	$\alpha_0 = 0$	$a_0 = 0$	$d_1 = 0$
2	${}^{1}_{2}T$	θ_2	$\alpha_1 = -90$	$a_1 = 0$	$d_2 = 0$
3	${}^{2}_{3}T$	θ_3	$\alpha_2 = 0$	a_2	d_3
4	${}^{3}_{4}T$	θ_4	$\alpha_3 = -90$	a_3	d_4
5	${}^{4}_{5}T$	θ_5	$\alpha_4 = 90$	$a_4 = 0$	$d_5 = 0$
6	${}^{5}_{6}T$	θ_6	$\alpha_5 = -90$	$a_5 = 0$	$d_6 = 0$

PUMA-560 Manipulator Kinematics

- **Now compute each of the link *transformations*:**

$$ {}^{0}_{1}T = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{1}_{2}T = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S\theta_2 & -C\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{2}_{3}T = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_2 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{4}T = \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -S\theta_4 & -C\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{4}_{5}T = \begin{bmatrix} C\theta_5 & -S\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S\theta_5 & C\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{5}_{6}T = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S\theta_6 & -C\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{i-1}_{i}T = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -S\alpha_{i-1} d_i \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & C\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

PUMA-560 Manipulator Kinematics

- **Let us now form the** ${}^{0}_{6}T$ ***transformation matrix*:**

$$ {}^{0}_{6}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T\,{}^{5}_{6}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

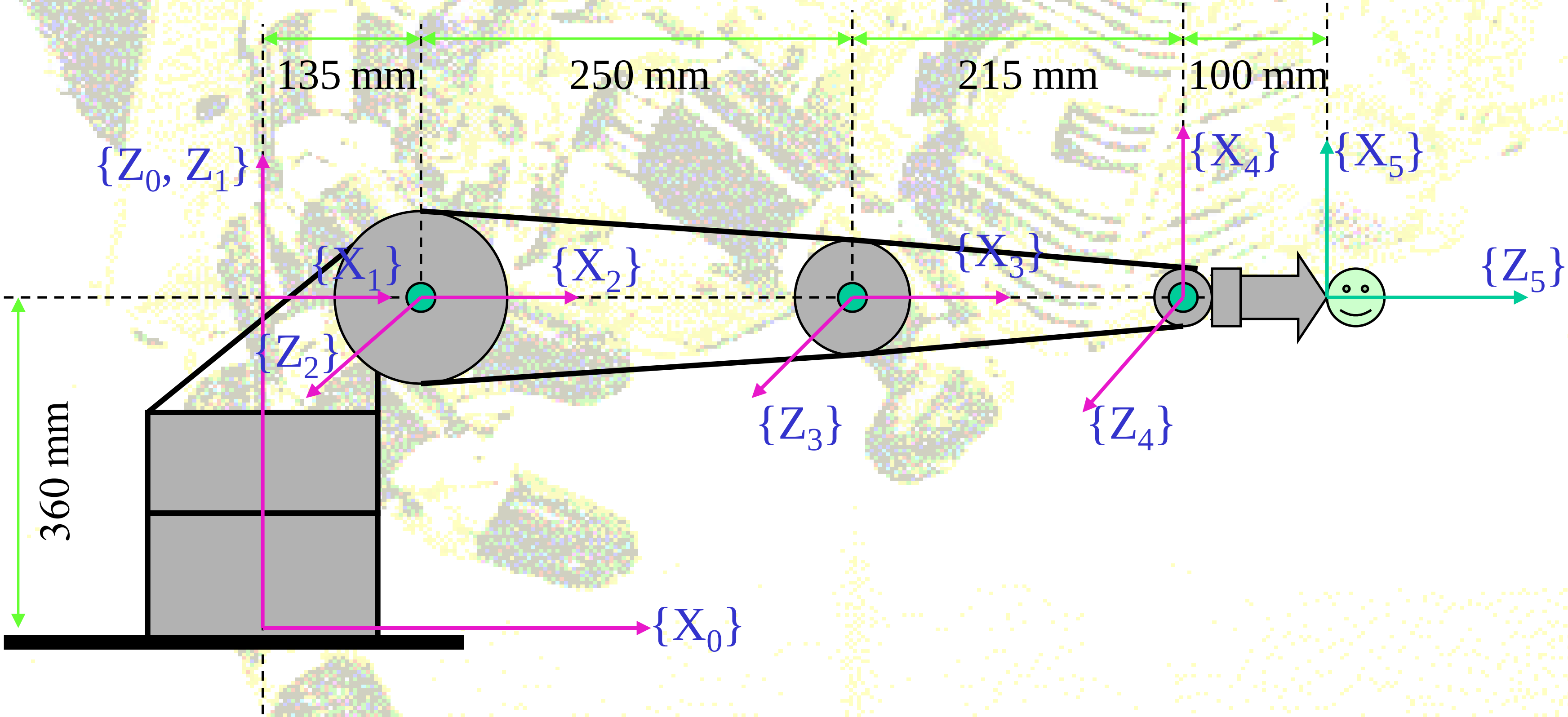
$$ \begin{aligned} r_{11} &= C_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] + S_1(S_4C_5C_6 + C_4S_6), \\ r_{21} &= S_1[C_{23}(C_4C_5C_6 - S_4S_6) - S_{23}S_5C_6] - C_1(S_4C_5C_6 + C_4S_6), \\ r_{31} &= -S_{23}(C_4C_5C_6 - S_4S_6) - C_{23}S_5C_6, \\ &\ldots \\ &\ldots \qquad Equation: (3.14) \end{aligned} $$

Note: In multiplying transformation matrices, when we have two neighboring parallel axes, one can use the sum of angle formulas to produce simpler expressions.

$$ S_{23} = S(\theta_2 + \theta_3) = C_2S_3 + S_2C_3 $$

Manipulator Kinematics

- **Example:** **The Yasukawa/Motoman MK3 Robot.**
  - **A 5-DOF "5R" Revolute Robot**



Manipulator Kinematics

- **The Yasukawa/Motoman MK3 Table of Link-Joint Parameters:**

Joint-i	${}^{i-1}_{i}T$	θ_i	α_{i-1}	a_{i-1}	d_i
1	${}^{0}_{1}T$	θ_1	$\alpha_0 = 0$	$a_0 = 0$	$d_1 = 360$
2	${}^{1}_{2}T$	θ_2	$\alpha_1 = 90$	$a_1 = 135$	$d_2 = 0$
3	${}^{2}_{3}T$	θ_3	$\alpha_2 = 0$	$a_2 = 250$	$d_3 = 0$
4	${}^{3}_{4}T$	θ_4	$\alpha_3 = 0$	$a_3 = 215$	$d_4 = 0$
5	${}^{4}_{5}T$	θ_5	$\alpha_4 = 90$	$a_4 = 0$	$d_5 = 100$

Yasukawa/Motoman MK3 Manipulator Kinematics

- **Now compute each of the link *transformations*:**

$$ {}^0_1T = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1_2T = \begin{bmatrix} C_2 & -S_2 & 0 & a_1 \\ 0 & 0 & -1 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2_3T = \begin{bmatrix} C_3 & -S_3 & 0 & a_2 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^3_4T = \begin{bmatrix} C_4 & -S_4 & 0 & a_3 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^4_5T = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S_5 & C_5 & 0 & -d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

YM MK3 Manipulator Kinematics

- **Let us now form the** $^{0}_{5}T$ ***transformation matrix*:**

$$^{0}_{5}T = {}^{0}_{1}T\,{}^{1}_{2}T\,{}^{2}_{3}T\,{}^{3}_{4}T\,{}^{4}_{5}T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$r_{11} = C_1 C_{234} C_5 + S_1 S_5 \qquad r_{12} = -C_1 C_{234} S_5 + S_1 C_5 \qquad r_{13} = C_1 S_{234}$$

$$r_{21} = S_1 C_{234} C_5 - C_1 S_5 \qquad r_{22} = -S_1 C_{234} S_5 - C_1 C_5 \qquad r_{23} = S_1 S_{234}$$

$$r_{31} = C_5 S_{234} \qquad r_{32} = -S_5 S_{234} \qquad r_{33} = -C_{234}$$

$$p_x = C_1 (a_1 + a_2 C_2 + a_3 C_{23} + d_5 S_{234})$$

$$p_y = S_1 (a_1 + a_2 C_2 + a_3 C_{23} + d_5 S_{234})$$

$$p_z = (d_1 + a_2 S_2 + a_3 S_{23} - d_5 C_{234})$$

Chapter 3 Exercises:

- 3.1, 3.3, 3.8, 3.9
- 3.1 Programming Exercise
- 3.1 MathLab Exercise
- Programming of the PUMA 560 Forward Kinematics